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A hybrid wavelet-based adaptive immersed boundary finite-difference lattice Boltzmann method for two-dimensional fluid–structure interaction

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A B S T R A C T

A second generation wavelet-based adaptive finite-difference Lattice Boltzmann method (FD-LBM) is developed in this paper. In this approach, the adaptive wavelet collocation method (AWCM) is firstly, to the best of our knowledge, incorporated into the FD-LBM. According to the grid refinement criterion based on the wavelet amplitudes of density distribution functions, an adaptive sparse grid is generated by the omission and addition of collocation points. On the sparse grid, the finite differences are used to approximate the derivatives. To eliminate the special treatments in using the FD-based derivative approximation near boundaries, the immersed boundary method (IBM) is also introduced into FD-LBM. By using the adaptive technique, the adaptive code requires much less grid points as compared to the uniform-mesh code. As a consequence, the computational efficiency can be improved. To justify the proposed method, a series of test cases, including fixed boundary cases and moving boundary cases, are invested. A good agreement between the present results and the data in previous literatures is obtained, which demonstrates the accuracy and effectiveness of the present AWCM-IB-LBM.

1. Introduction

As an alternative to the traditional Navier–Stokes (N-S) equation solver, the lattice Boltzmann method (LBM) has been used popularly in the simulation of fluid flow problem over the last two decades [1]. Compared to the traditional numerical methods, the LBM is very new. Its noticeable advantages, such as simplicity, parallelizability and explicit calculation lead to its great popularity. With years of continuous improvement, the LBM has been successfully used in many different areas, such as flow dynamics, noise control problems and fluid–structure interaction problems [2–5].

Based on the kinetic theory, the LBM use the density distribution functions to describe the fictitious particles. The dynamic of the fictitious particles is studied by the evolution of the density distribution functions. For the standard LBM, the whole calculation process can be made up by two sub-processes, i.e. the collision and the streaming. The macroscopic variables, such as density and velocity, can be calculated by the distribution functions. In the standard LBM, it is required that particles should move from one mesh point to its neighbor points within a time interval in the streaming sub-process. So the calculation mesh is limited to uniform mesh. Because of this mesh requirement, the application of the LBM is limited

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greatly. To eliminate this shortcoming, great improvements have been made by many researchers to implement the LBM on the non-uniform mesh. The strategies can be roughly classified into two types: interpolation type and differential type [6].

For the first type, He and his co-workers [7,8] firstly introduced an interpolation step into the LBM and proposed the interpolation-supplemented lattice Boltzmann method (ISLBM). Based on the ISLBM, some other interpolating type LBM were proposed. By proposing a generalized form of the interpolation supplemented LBM (GILBM), Imamura et al. [9] make the ISLBM applicable to arbitrarily structured mesh. And by using the local time step, the GILBM has a very high efficiency. Based on the Taylor series expansion and least squares theory, Shu et al. [10,11] proposed the Taylor series expansion and least squares-based lattice Boltzmann method (TLLBM). This method has been successfully used to simulate flows with complex geometry. For the second type, by using Taylor series expansion, the lattice Boltzmann equations (LBEs) are transformed into a set of differential equations. These differential equations can be solved by the conventional numerical methods such as finite difference (FD), finite element (FE), and finite volume (FV). Perhaps, the finite difference LBM (FDLBM) was firstly proposed by Reider and Sterling [12], and then was examined by Cao et al. [13]. Benefiting from the work of Mei and Shyy [14], the FDLBM can be implemented on the curvilinear coordinates. By using a predictor-corrector method, Lee and Lin [15] proposed the finite-element LBM (FELBM). And then, some other finite element-based lattice Boltzmann methods were proposed [16,17]. By introducing the FVM to solve the LBMs, Succi et al. [18] firstly proposed the finite volume LBM (FVLBM). After that, based on the cell-centered data structure-based FVM, the FVLBM were implied on the structured mesh [19]. And based on the vortex-centered data structure-based FVM, the FVLBM has been successfully implied on the unstructured mesh [20,21].

There are many adaptive-mesh-refinement (AMR) techniques used in the FD, FE and FV. By introducing these techniques into the LBM, an adaptive lattice Boltzmann can be proposed. J. Wu and C. Shu [6] proposed a solution-adaptive LBM by introducing an efficient stencil adaptive technique, originally proposed by Ding and Shu [22], into the ISLBM. In the method, by coupling 5-point orthogonal symmetric stencil and 5-point diagonal symmetric stencil, a 9-point symmetric structure can be generated, which is similar to the D2Q9 lattice model. Guzik et al. [23] proposed a novel adaptive FVLBM. In the adaptive FVLBM, the key point is transferring information across interfaces between different grid resolutions. This key point was solved by a new method which was developed following established techniques for finite-volume representations. On the basis of a block-structured AMR, A. Fakhari and T. Lee [24] proposed an adaptive finite-difference lattice Boltzmann method. By using pointers, there is no need to search all blocks to find the neighboring blocks of a given block in this method. Another highlight is that the use of a special finite-difference lattice Boltzmann scheme which is following the idea proposed in [25] ensures that all of the blocks at different refinement levels can be advanced in time with the same time step. More recently, based on a “bubble” function which can eliminate the need of time interpolation, Guo et al. [26] proposed a hybrid adaptive LBM to simulate two-dimensional flow. It is obvious that incorporating the AMR techniques, used in the FD, FE and FV, into the LBM for more efficient and effective adaptive method attracts increasing attention.

In this paper, the Adaptive Wavelet Collocation Method (AWCM) is firstly, to the best of our knowledge, incorporated into FDLBM to develop a new adaptive LBM. The AWCM is based on the second generation wavelet transformation. By being wavelet transformed, a function's local regularity can be reflected by wavelet amplitudes. In the rapidly varying regions, wavelet amplitudes are relatively large. While in the regions where the solution is well-behaved, the wavelet amplitudes are small. So in the regions where wavelet amplitudes are small, there is no need to use such a fine grid as in the regions where wavelet amplitudes are large. By omitting points with small wavelet amplitudes, as well as the wavelet transformation corresponding to these points, a sparse grid can be generated. And this sparse grid is dynamically adapted to evolving features of the solution [27]. Besides, the partial differential equations (PDEs) are solved in the physical space in the AWCM which is not the same as in the other transform-based methods such as Fourier transform-based method and adaptive wavelet Galerkin method. As a result, the derivations can be conveniently and efficiently approximated by using finite-differences, which is proposed by O.V. Vasilyev and C. Bowaman [28]. The details of this procedure and the error bound of this finite-differences based derivation approximation are given in Ref. [28]. Since it was proposed, the AWCM has been a very useful solver for PDEs such as parabolic [28], hyperbolic [29] and elliptic [30]. It has been also applied to various engineering areas by many researchers [27,29,31–33]. In this paper, the AWCM is incorporated into the FDLBM proposed by T. Lee and C.-L. Lin [25]. Based on the grid refinement criterion which is proposed in this paper and explained in the following text, the adaptive omission and addition of computing points are conducted. As a result, a sparse grid is generated. The computing procedures are conducted on this sparse grid. For the computing grid to be compressed, the computing resources can be saved and computing efficiency can be improved.

As is well known, some special treatments are needed in using FD-based derivation approximation near the boundaries. Even, special treatments will be more complex if there are one or more boundaries in the inner field of the calculation domain. And the AWCM is not suitable for the complex body-fitted mesh. So if the inner boundaries can be eliminated, the use of FD-based derivation approximation and AWCM will be more convenient and the computing efficiency will be also improved. Fortunately, the immersed boundary method (IBM) proposed by Peskin [34] can eliminate the need of these special treatments. In the IBM, a fixed Cartesian mesh is used to represent the flow field, and a Lagrangian mesh is used to represent the boundary. The discrete delta function interpolation is used to relate the variables on the two meshes to each other. As the brightest spot of the IBM, the effect of boundary is reflected by the restoring force acting on the Eulerian mesh in the vicinity of boundary. The N-S equations with the restoring force are solved over the whole Eulerian domain. So there is no inner boundary in solving the N-S equations, which is very suitable for the implementation of the AWCM and the FD-based derivation approximation. The IBM can be used to solve not only the fixed boundary problems but