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An improved imaging method for extended targets

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ABSTRACT

When we use direct imaging methods for solving inverse scattering problems, we observe artificial lines which make it hard to determine the shape of targets. We propose a signal space test to study the cause of the artificial lines and we use multiple frequency data to reduce the effect from them. Finally, we use the active contour without edges (ACWE) method to further improve the imaging results. The final reconstruction is accurate and robust. The computational cost is lower than iterative imaging methods which need to solve the forward problem in each iteration.

1. Introduction

There are many important applications based on direct and inverse scattering problems, for example, in radar, sonar, medical imaging, and nondestructive testing. By sending incident waves, with the information of the targets, we could measure the scattered waves. By sending incident waves and collecting the scattered waves, we could identify the location and shape of targets. There are some early work on point targets [21–23,26] or small targets [20] compared with the array resolution. We are focusing on the more challenging case of extended targets. For direct scattering problems, if we view the total field as the sum of an incident field u^{i} and a scattered field u^{s}, then the problem is to determine the u^{s} from the u^{i} and the differential equation which is the Helmholz equation with proper boundary conditions. The boundary integral method in [9] can be applied to solve such problem. For inverse scattering problems, there are two main ways to solve the problem: iterative methods and direct methods. The iterative methods [1–7,10,27] are accurate but more expensive, and the direct methods [8,11–13,17,24,25] are efficient but less accurate. However, we are not focusing on the iterative methods, because for each iteration, it costs too much time to solve an adjoint forward problem.

These days direct imaging methods are more popular. They are not based on nonlinear optimization and do not require forward iterations, for example, the MUltiple SIgnal Classification (*MUSIC*) algorithm [21–23]. We first set up the response matrix; then we could obtain the singular values and singular vectors by taking the singular value decomposition (*SVD*) of the response matrix. By using the imaging function involving Green's function, the inverse scattering problem can be solved.

The linear sampling method [24,28] is also a direct imaging algorithm for the inverse scattering problem. It characterizes the domain of an unknown scatterer by the behavior of the solution to an integral equation of the first kind. The main idea is that the norm of a certain solution blows up on the target boundary. Kirsch [25] modified the linear sampling method by using a factorization for the scattering operator. It appears that the linear sampling method is an extension of the *MUSIC* algorithm [14], so it can also produce the location and shape of the target by using all the eigenvalues and eigenfunctions.

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We refer to [18] for the uniqueness in inverse acoustic and electromagnetic obstacle scattering problems.

When we use the *MUSIC* algorithm or other direct imaging methods to obtain the shape of the target, we observe many artificial lines. We wonder how the artificial lines form, and how we can avoid them. Hence, we propose a signal space test to study the cause of the artificial lines. In order to decrease the effect of these lines, we combine multiple frequency results. We use both the multiple frequency *MUSIC* algorithm and the multi-tone method [12,13].

Finally, we proposed a novel method by using the Active Contour Without Edges (*ACWE*) method [15] to improve the imaging results. Even though this method has existed in the literature for decades, it has not been applied to improve inverse problem solvers. The idea is that the average location of the two sharp gradient lines is a reasonable estimate for the target shape. Numerical examples for smooth target, for targets with one corner and for multiple corners are presented with excellent results. These results are more accurate than those obtained from direct imaging methods, such as the *MUSIC* algorithm [11,12], the multi-tone algorithm [13] and the linear sampling method [24,25]. This is because other methods typically have a more blurred target boundary. Compared with iterative methods [1], our method shares the feature that curve evolution is involved. However, our procedure does not involve solving the forward problem, while iterative methods have to solve the forward problem in each iteration. Therefore, our method is both efficient and accurate. Further, numerical experiments show that it is robust with respect to noisy data.

2. The forward problem

Consider the Helmholtz equation [9] with the Dirichlet boundary condition:

$$\begin{aligned} \Delta u + k^2 u = 0 \quad &\text{in} \quad \mathbb{R}^2 \setminus \bar{D}, \\ u = 0 \quad &\text{on} \quad \partial D, \end{aligned} \tag{1}$$

where k is the wavenumber.

The incident field $u^{\mathrm{i}} = e^{ik\mathbf{x}\cdot\mathbf{d}}$, where $\mathbf{d} \in \mathbb{S}^1$ is the incident direction, satisfies the homogeneous equation:

$$\Delta u^{\mathrm{i}} + k^2 u^{\mathrm{i}} = 0 \quad \text{in} \quad \mathbb{R}^2 \setminus \bar{D} \tag{2}$$

The total field consists of the incident field u^{i} and the scattered field u^{s}

$$u = u^{\mathrm{i}} + u^{\mathrm{s}}. \tag{3}$$

From Equation (1), (2), (3), we know that the scattered field satisfies

$$\Delta u^{\mathrm{s}} + k^2 u^{\mathrm{s}} = 0 \quad \text{in} \quad \mathbb{R}^2 \setminus \bar{D}, \tag{4}$$

$$u^{\mathrm{s}} = -u^{\mathrm{i}} \quad \text{on} \quad \partial D. \tag{5}$$

$$\lim_{r\to\infty} \sqrt{r}\left(\frac{\partial u^{\mathrm{s}}}{\partial r} - iku^{\mathrm{s}}\right) = 0 \quad r = |\mathbf{x}|, \tag{6}$$

Equation (6) is called the Sommerfeld radiation condition, and the limit is assumed to hold uniformly in all directions $\mathbf{x}/|\mathbf{x}|$. Equation (6) is imposed for u^s for the physical meaning and it also guarantees the uniqueness result for the forward scattering problem.

We now briefly go through the layer approach for finding a solution to the Dirichlet problem using boundary integral equations [9]. We need to use a combined double and single layer potential. The equation of the smooth target case is as follows:

$$u(x) = \int_{\partial D} \left\{ \frac{\partial \Phi(x,y)}{\partial \nu(y)} - i\eta\Phi(x,y) \right\} \varphi(y) ds(y), \quad x \in \mathbb{R}^2 \setminus \partial D, \tag{7}$$

with a density $\varphi \in C(\partial D)$ and the parameter $\eta \neq 0$. Φ is the fundamental solution to the Helmholtz equation. $\Phi(x,y) = \frac{i}{4} H_0^1(k|x-y|)$, where H is the Hankel function. ν is the unit outer normal.

Note that if $\eta = 0$, the problem is not uniquely solvable. From the jump relations [9], we have the integral equation:

$$\varphi(x) + 2\int_{\partial D} \left\{ \frac{\partial \Phi(x,y)}{\partial \nu(y)} - i\eta\Phi(x,y) \right\} \varphi(y) ds(y) = 2f(x), \quad x \in \partial D \tag{8}$$

where f is negative of the incident wave u^i.

Then Nystrom discretization in [9] is applied to convert the above integral equation to a linear system, so that the approximation for the density function φ can be computed.

With the fundamental solution to the Laplace equation $\Phi_0(x,y) := \dfrac{1}{2\pi} \ln \dfrac{1}{|x-y|}$, where $x \neq y$, we rewrite the equation (7) for the case with one corner: