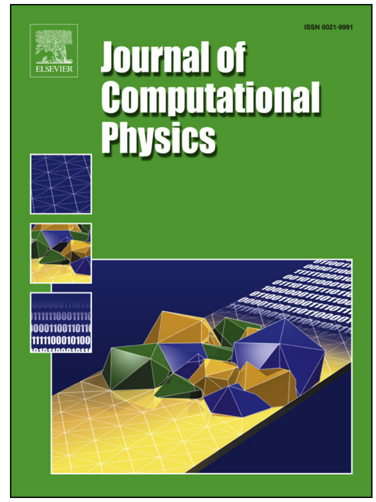

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Che Wang, Hong Wang

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Che Wang Hong Wang

Department of Mathematics, University of South Carolina, Columbia, South Carolina 29208, USA (e-mail address: hwang@math.sc.edu, telephone: 803-777-4321. fax: 803-777-6527)

Abstract

We develop a fast collocation scheme for a variable-coefficient nonlocal diffusion model, for which a numerical discretization would yield a dense stiffness matrix. The development of the fast method is achieved by carefully handling the variable coefficients appearing inside the singular integral operator and exploiting the structure of the dense stiffness matrix. The resulting fast method reduces the computational work from $O(N^3)$ required by a commonly used direct solver to $O(N \log N)$ per iteration and the memory requirement from $O(N^2)$ to $O(N)$. Furthermore, the fast method reduces the computational work of assembling the stiffness matrix from $O(N^2)$ to $O(N)$. Numerical results are presented to show the utility of the fast method.

Keywords: fast collocation method, fractional diffusion equation, nonlocal diffusion model, nonlocal model

1. Introduction

Nonlocal models provide an alternative means to fractional partial differential equations [2, 9, 13, 16, 17] in modeling challenging phenomena such as anomalous diffusion and long-range spatial interactions, which cannot be modeled properly by conventional integer-order differential equations [7, 8, 11]. However, its numerical discretizations yield dense stiffness matrices, for which commonly used direct solvers require $O(N^2)$ memory storage and $O(N^3)$ computational cost where N is the number of unknowns. These significantly increased computational complexity and memory requirement becomes one of the main obstacles that hinders its applications.