



Displacement data assimilation

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ABSTRACT

We show that modifying a Bayesian data assimilation scheme by incorporating kinematically-consistent displacement corrections produces a scheme that is demonstrably better at estimating partially observed state vectors in a setting where feature information is important. While the displacement transformation is generic, here we implement it within an ensemble Kalman Filter framework and demonstrate its effectiveness in tracking stochastically perturbed vortices.

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1. Bayesian estimation and displacement assimilation

Most sequential estimation strategies are Bayesian. They seek to find estimates of moments of the posterior probability density function of the random, time-dependent, state variable $X(t)$, subject to noisy observations of the state, $Y(t)$. The posterior probability is thus

$$P(X|Y) \propto P(Y|X)P(X),$$

where the first term on the right hand side is the likelihood, and is informed by observations. $P(X)$ is the prior, which is informed by the model. The model is usually an evolution equation for X . The model “error” might be an explicit stochastic term representing uncertainties in the evolution equation or its initial or boundary data. The observations are usually stochastic because there are inherent errors in the measurements. When the prior and the likelihood are Gaussian (or a product of Gaussians), minimizing the sum of the quadratic arguments of $P(Y|X)$ and $P(X)$ (equivalently $\log P(Y|X) + \log P(X)$) maximizes $P(X|Y)$. Moreover, it does so by taking into account the relative certainties in the model and in the measurements. The most familiar sequential estimation technique is the Kalman Filter (see [1]). It seeks a minimizer of the trace of the posterior covariance. It yields an optimal estimate of the time dependent posterior mean and covariance of $X(t)$, over some interval in t when the stochasticity is normally distributed, the dynamics are linear, and relationship between the observations and the state is a linear transformation.

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The extended Kalman Filter (see [1]) and the ensemble Kalman Filter (enKF) [2] are frequently-used in mildly non-linear problems with inherently modal statistical distributions. They are not guaranteed to converge without exceptions. The enKF is very popular since it is relatively easy to implement. It will be employed in this study, as it provides a useful and familiar framework for developing more targeted assimilation methods. The enKF strategy is, like the Kalman Filter approach, a two-stage sequential estimation process. There is a forecast, wherein the model is advanced from $t - \delta t$ to t . An ensemble forecast is computed (the model is usually not linearized) by perturbing slightly the state at $t - \delta t$. This is then followed by an analysis stage, which is the Gaussian update from the Kalman scheme. In the absence of measurements, the best estimate is the sample mean and covariance of the ensemble forecast. (This presumes very critically that the mean of forecast errors remains close to zero.) If observations are available at that time t , these are individually assimilated into each ensemble member. Rather than solving the update of the covariance matrix, one uses a sample covariance based on the ensemble of forecasts. More details on the enKF filtering scheme appear in [Appendix B](#).

Most data assimilation schemes are variance-minimizing; they give an appropriately weighted average of the predictions of the model and the noisy observations [3]. These methods thus tend to decrease sharp gradients, consequently smearing or obliterating “features” (vortices, shock-fronts, etc.) in the state variable being estimated. If tracking the features are critical, a purely variance-minimizing methodology will produce estimates that might not be accurate enough for prediction, particularly in problems where capturing characteristics is critical, as in wave propagation problems. In this paper we propose a modification to sequential state estimation that can improve estimates where features are important. We denote this two-state data assimilation strategy *displacement assimilation*. Many of the sequential data assimilation procedures yield an estimate of $X(t)$ that may not be in the solution space of the model; for that matter, it might not even be physical. Constraints may be added to the Bayesian statement to promote physically reasonable properties in the analysis estimate, which motivates inserting an extra step into the data assimilation process. The method developed here applies this strategy to improve the estimates of a variance minimizing strategy when morphological features in the state variable are important. For example, suppose we want to estimate characteristic paths for a wave process. These paths are both space and time dependent (phase dependent). If designed properly, an assimilation method that makes a phase correction in addition to a blending of observations and model outcomes might deliver better estimates of such things, as the space–time information of what generated a wave, or the space–time information that better tracks the wave characteristics. This two-stage process is what we call *displacement assimilation*.

If the displacement correction and the estimation process are kept separate, it is possible to develop a displacement correction scheme that could then be applied to a variety of different estimation strategies. The development we present here is in that spirit. In this study we will be contrasting the standard enKF and something we call displacement enKF, wherein the difference is that in the latter we add a phase correction.

Adding phase corrections within the context of data assimilation is not a new idea. Among other works, we can mention [4], who argued how this procedure might improve predictions in meteorology (see also [5], [6]). Ravella and collaborators [7] applied variational techniques (3DVAR) to jointly optimize displacement and amplitude corrections, and improve alignment of vortex-like features. Recently, displacement assimilation has been incorporated into the spatio-temporal data assimilation capabilities (4DVAR) of the weather forecasting tool WRF DAS for short-term hurricane prediction [8].

Percival used ideas from control theory to find coordinate transformations that could improve predictions [9]. He applied area preserving flows whose limit in a pseudo-time variable defines a phase correcting map that locally minimizes a Bayesian-type penalty functional. In contrast, we will use area preserving maps to do the position correction “in one shot” every time new data is assimilated into the model.

An additional criterion in our methodology is to ensure that the phase corrections are kinematically consistent with the underlying physics of the system. This is not always necessary for making phase corrections. Frazin [10], for example, makes phase corrections that have no physical basis – the aim of his work is to improve the optical data using a data assimilation system. In our context however, it is critical that the corrections be consistent with the underlying physics, *because we are using the phase corrections to capture the effects of unresolved/unmodeled (but physically constrained) effects in the system*.

The key in regularizing (or imposing) convexity in the space of displacement maps is to invoke physics or some ad-hoc constraint. We suggest the use of a physically-motivated constraint. When compared to an ad-hoc constraint, a physics-motivated constraint leads to estimates whose optimality can be understood on physical grounds. Intuitive approaches to displacement corrections can be found in early works by Brewster [5,6]. This strategy is formalized mathematically by Ravella [7,11], who proposes that the optimal map be constructed so as to minimize the wave number of the displacement. The present work proposes an alternative form and implementation of these constraints, pursuing the same motivation.

2. Displacement assimilation, defined

Consider the problem of determining the state y which best explains a set of noisy observations $d = h(y^t) + \epsilon$ of an unknown true state y^t , where the nonlinear observation operator can be modeled by $h(y)$ and the observation noise $\epsilon \sim \mathcal{N}(0, R)$. We want to solve the ill-posed problem $d = h(y)$, and resort to minimizing the likelihood functional,

$$\mathcal{J}_L[y] = \|d - h(y)\|_{R^{-1}}^2 \quad (1)$$

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