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A two-stage adaptive stochastic collocation method on nested sparse grids for multiphase flow in randomly heterogeneous porous media

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ABSTRACT

A new computational method is proposed for efficient uncertainty quantification of multiphase flow in porous media with stochastic permeability. For pressure estimation, it combines the dimension-adaptive stochastic collocation method on Smolyak sparse grids and the Kronrod–Patterson–Hermite nested quadrature formulas. For saturation estimation, an additional stage is developed, in which the pressure and velocity samples are first generated by the sparse grid interpolation and then substituted into the transport equation to solve for the saturation samples, to address the low regularity problem of the saturation. Numerical examples are presented for multiphase flow with stochastic permeability fields to demonstrate accuracy and efficiency of the proposed two-stage adaptive stochastic collocation method on nested sparse grids.

1. Introduction

There has been considerable interest in studying subsurface flow problems with stochastic inputs in the past few decades [1–3]. These stochastic inputs may arise from the random heterogeneities of porous media or from the uncertainties in boundary and initial conditions [4,5]. The former heterogeneities are usually difficult to quantify because of the incomplete knowledge of the properties, such as permeability, at each location in the domain. In many cases, these properties are viewed as random fields that satisfy some statistical descriptions [6,7]. This randomness hence leads to uncertain model outputs, such as pressure, velocity, and saturation.

Intensive research has been performed in quantifying the uncertainty of the model outputs. The Monte Carlo (MC) method is probably the best-known method, which is easy to implement and naturally decoupled. However, it usually requires a large number of realizations and could be computationally prohibitive. Instead, the stochastic collocation method (SCM), based on sparse grids, serves as an efficient alternative [8–12]. This method represents the stochastic output as a polynomial approximation, and calculates the integration and/or constructs the interpolation based on the Smolyak algorithm. It is parallelizable and exhibits a fast convergence rate, provided that the output is sufficiently smooth [13,14].

However, there are several challenges to applying the SCM to multiphase flow problems. First, the number of collocation points increases dramatically as the random dimension increases, i.e., the so-called curse of dimensionality. In fact, the convergence rate could be so slow that a high level of accuracy cannot be obtained even for a moderate number of

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dimensions. One approach to alleviate the computational cost is to use adaptivity. Gerstner and Griebel [15] developed a dimension-adaptive quadrature approach by searching for important dimensions automatically and placing more collocation points in those dimensions. Klimke [16] extended this approach from quadrature to interpolation and applied it to the fuzzy set-based uncertainty modeling process. Ganapathysubramanian and Zabaras [17] introduced an adaptive refinement in important stochastic dimensions to reduce numerical effort. Nobile et al. [18] proposed an anisotropic sparse grid collocation method and provided a rigorous convergence analysis. However, these studies assumed uniformly distributed random variables, while the log-permeability of the porous media is usually considered as Gaussian random fields [1].

Second, the number of collocation points increases rapidly if the univariate quadrature nodes are unnested. Therefore, nested nodes are desired for efficiency. For uniform distribution, the unnested Gauss–Legendre nodes [19] can be replaced by nested Kronrod–Patterson–Legendre nodes [20] or Clenshaw–Curtis nodes [21]. Regarding normal distribution, the unnested Gauss–Hermite (GH) nodes [19] can be replaced by the nested Kronrod–Patterson–Hermite (KPH) nodes [22,23]. However, to the best of our knowledge, the KPH nodes have not been used for stochastic analysis of multiphase flow problems.

Third, the convergence rate of the SCM depends on the regularity of the model output with respect to the random input. As is well known, the global polynomial interpolation cannot resolve local discontinuity in the stochastic space. For example, the phase saturation in multiphase flow is discontinuous in physical space if a shock front exists. In this case, the estimated saturation samples from the SCM could be oscillatory and thus unphysical. A similar phenomenon is observed in the solute transport problems [24]. A variety of approaches are proposed to address these kinds of difficulties. For example, Foo et al. [25,26] formulated a collocation method to discretize the random space of inputs into multiple subspaces. Le Maître et al. [27] presented a wavelet-based method with localized decompositions. Ma and Zabaras [28,29] presented a method that refines the sparse grids locally by working directly in the hierarchical basis. These methods may capture local discontinuity, but at the cost of more collocation points in the sub-domains, and thus a slower convergence rate. Liao and Zhang [30–32] proposed a series of transformed approaches, which approximate some alternative variables (e.g., location, displacement or arrival time) instead of the outputs (e.g., phase saturation or solute concentration) by polynomials. However, these approaches possess certain limitations, e.g., the displacement-based transformed approach requires topology similarity of the outputs in different realizations.

In this study, an adaptive stochastic collocation method on Kronrod–Patterson–Hermite sparse grids (ASCM–KPH) is proposed, which combines the dimension-adaptive approach and the nested nodes, to quantify the uncertainty of the pressure and velocity for the multiphase flow problems. Regarding saturation, an additional stage is introduced, in which the pressure and velocity samples are first generated by the polynomial interpolation and then substituted into the transport equation to solve for the saturation samples, which are always physically meaningful.

This paper is organized as follows: In Section 2, the governing equations of multiphase flow in porous media are introduced. In Section 3, a mathematical framework of the proposed method is formulated. In Section 4, the proposed method is tested in numerical examples. Finally, concluding remarks are provided in Section 5.

2. Governing equations

The oil/water two-phase immiscible flow model can be expressed by the following continuity equation as [33]:

$$\frac{\partial(\phi\rho_\alpha S_\alpha)}{\partial t} = -\nabla\cdot(\rho_\alpha \mathbf{u}_\alpha) + q_\alpha, \quad \alpha = w, o, \tag{1}$$

where ϕ is the porosity; and each phase has its own density ρ_α, saturation S_α, phase velocities $\mathbf{u}_\alpha$, and source term q_α. Darcy's law for multiphase flow is:

$$\mathbf{u}_\alpha = -\frac{\mathbf{k}k_{r\alpha}}{\mu_\alpha}(\nabla p_\alpha - \rho_\alpha g \nabla z), \quad \alpha = w, o, \tag{2}$$

where $\mathbf{k}$ is the absolute permeability; g is the gravitational acceleration; z is the depth; and $k_{r\alpha}$, μ_α, and p_α are the relative permeability, viscosity, and pressure for phase α, respectively. Eqs. (1) and (2) are also referred to as the transport equation and the flow equation, respectively [34]. They are usually coupled with:

$$S_w + S_o = 1, \quad p_c(S_w) = p_o - p_w, \tag{3}$$

where p_c is the capillary pressure; which is a function of S_w. In this study, the absolute permeability is considered as a random field, and the statistics of pressure and saturation are estimated.

3. Methodology

3.1. Stochastic collocation method (SCM)

The stochastic collocation method (SCM) builds interpolation functions for the model outputs using their values at particular collocation points in the stochastic input space. In most problems, the ultimate goal is usually to obtain the moments and the probability distribution functions (PDFs) of the model outputs. This is accomplished through specific quadrature