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Multidimensional Hall magnetohydrodynamics with isotropic or anisotropic thermal pressure: Numerical scheme and its validation using solitary waves

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A R T I C L E I N F O

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A B S T R A C T

We present a numerical solver for plasma dynamics simulations in Hall magneto-hydrodynamic (HMHD) approximation in one, two and three dimensions. We consider both isotropic and anisotropic thermal pressure cases, where a general gyrotropic approximation is used. Both explicit energy conservation equation and general polytropic state equations are considered. The numerical scheme incorporates second-order Runge–Kutta advancing in time and Kurganov–Tadmor scheme with van Leer flux limiter for the approximation of fluxes. A flux-interpolated constrained-transport approach is used to preserve solenoidal magnetic field in the simulations. The implemented code is validated using several test problems previously described in the literature. Additionally, we propose a new validation method for HMHD codes based on solitary waves that provides a possibility of quantitative rigorous testing in nonlinear (large amplitude) regime as an extension to standard tests using small-amplitude whistler waves. Quantitative tests of accuracy and performance of the implemented code show the fidelity of the proposed approach.

1. Introduction

Hall magnetohydrodynamics (HMHD hereafter) provides a natural extension of ideal or resistive magnetohydrodynamic (MHD) models for plasmas in the limit of small scales, where the magnetic field is frozen into electron fluid but ions are decoupled from the magnetic field lines [1]. As related to different masses of ions and electrons, the inertial effects become important at scales of the order of the ion inertial length (sometimes referred to as the ion skin depth) $d_i = V_A/\Omega_i$, where V_A is the Alfven speed and Ω_i is the ion gyrofrequency. The HMHD physics is essentially contained in the Ohm's law modified in comparison with the MHD formulation, which influences the transport of the magnetic field in plasma through the Faraday's induction equation. The Hall term also enters the energy conservation equation. Dispersive effects related to the Hall term are responsible for the appearance of so-called whistler waves. HMHD-related phenomena are studied as an important element of fast magnetic reconnection [2–4]. The HMHD physics includes also processes of formation of solitary waves [5–8]. The Hall term is also important for modeling small-scale fluctuations in plasma turbulence [9,10].

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In collisionless or weakly collisional plasmas one may expect the development of thermal pressure anisotropies. Lack of collisional mechanisms of exchange of particle energy between degrees of freedom parallel and perpendicular to the magnetic field direction may obviously lead to an asymmetric distribution function for particle velocities. In the lowest-order approximation, a gyrotropic model of anisotropy applies, where the distribution function is assumed to be bi-Maxwellian and axially symmetric with respect to the local magnetic field direction. In this approach, the parallel and perpendicular temperatures are in general different and they evolve in time in a different way. The pressure anisotropy is known to provide free energy for the development of instabilities, that are believed to control the pressure anisotropy in space plasmas as measured in-situ in the solar wind [11–14]. Questions related to the pressure anisotropy regulation in space plasmas have been investigated extensively in various astrophysical aspects [15–19].

There exist a number of numerical codes for numerical simulations within the HMHD framework. The codes use explicit time advancing (e.g. [20]) or implicit scheme (e.g. [21,22]). Efforts have been made towards including adaptive mesh refinement in HMHD simulations [23]. However, quantitative validation of HMHD codes in nonlinear regime is difficult due to the lack of analytic or semi-analytic problems that could be used for this purpose. Quantitative testing of the accuracy of HMHD codes consists mainly in studying of propagation of small-amplitude whistler waves in the computational domain. To our knowledge, no general method of testing of absolute accuracy has been proposed for the nonlinear regime of HMHD dynamics.

In this paper, we discuss a method of solving of the HMHD equations with the isotropic or anisotropic thermal pressure. The algorithm can be briefly described as using the second-order Runge–Kutta advancing in time and Kurganov–Tadmor scheme with van Leer flux limiter for the approximation of fluxes. To preserve solenoidal magnetic field during time evolution, the magnetic field transport equation is advanced in time using so-called flux-interpolated constrained-transport approach. The pressure tensor can be modeled in a gyrotropic approximation with polytropic relations describing the evolution of the parallel and perpendicular pressures. It is also possible to use an equation for the evolution of the perpendicular pressure and the explicit energy conservation equation, which guarantees the conservation of the total energy averaged over the simulation box to a very high accuracy. For isotropic pressure case also a polytropic state equation or the explicit energy conservation equation can be used. The presented scheme is intended for simulations of phenomena in the range of scales of the order of the ion inertial length and larger. This range of scales is determined by a general physical regime of validity of the HMHD equations, but also by the explicit character of the proposed numerical scheme that imposes strong constraints on the simulation time step. The algorithm is shown to work properly for one-, two- and three-dimensional test problems of different types: solitary waves propagation, magnetic reconnection, and the growth of the firehose instability. In this paper, we also discuss thoroughly a new testing method based on the propagation of solitary structures as a possible testing framework for HMHD in the nonlinear regime.

2. Physical model

2.1. HMHD equations in conservative form

The following equations can be derived as describing plasma dynamics on scales comparable to the ion inertial length scale in the collisionless plasma regime within fluid approximation (see, e.g. Refs. [1,7,24] for details). The mass and momentum transport can be calculated by the following equations

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}) \tag{1}$$

and

$$\rho \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} \right] = \mathbf{J} \times \mathbf{B} - \nabla \cdot \mathbb{P}, \tag{2}$$

correspondingly. The Ampere's law defines the current density $\mathbf{J} = \mu_0^{-1} \nabla \times \mathbf{B}$, $\mathbb{P}_{ij} = p_\perp \delta_{ij} + (p_\parallel - p_\perp) B_i B_j / B^2$ is the pressure tensor (gyrotropic approximation, $\parallel$ and $\perp$ directions are defined with respect to the local magnetic field direction), $\rho = N m_\mathrm{i}$ is the proton density, N is the proton number density, $\mathbf{u}$ is the plasma velocity vector, $\mathbf{B}$ is the magnetic field vector, m_i is the proton mass. The generalized Ohm's equation

$$-\mathbf{u}_\mathrm{H} \times \mathbf{B} = \mathbf{E} + \mathbf{u} \times \mathbf{B} - \eta \mathbf{J} \tag{3}$$

contains a Hall term on the left-hand side, where $\mathbf{u}_\mathrm{H} = -\mathbf{J}/eN$ is a Hall velocity vector, e is the proton charge. The resistive term $\eta \mathbf{J}$ allows to incorporate effects of finite resistivity in the model, where η formally denotes the magnetic diffusivity. The above equations can be obtained formally from the kinetic Vlasov equation using a standard procedure based on subsequent moments of the velocity distribution function, where all terms proportional to the electron inertial length are neglected [24]. Additionally, we assumed here a small electron temperature since otherwise an additional term proportional to the gradient of the electron pressure $\nabla p_e / eN$ would have been required in Eq. (3). The electron pressure term could be incorporated into the model in a simplified way (scalar pressure evolution by using isothermal or polytropic equation of state), but a more elaborated approach with anisotropy of the electron pressure is presumably advantageous at least for some problems, like