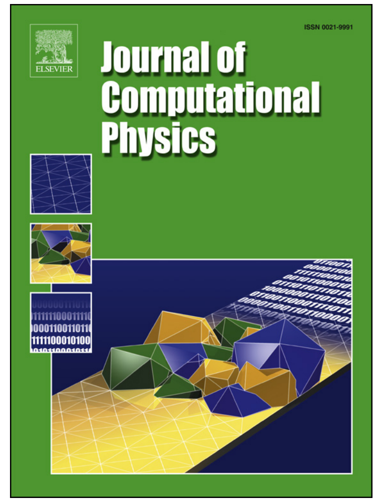

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Numerical solution of the homogeneous Neumann boundary value problem on domains with a thin layer of random thickness

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Abstract

The present article is dedicated to the numerical solution of homogeneous Neumann boundary value problems on domains with a thin layer of different conductivity and of random thickness. By changing the boundary condition, the boundary value problem given on the random domain can be transformed into a boundary value problem on a fixed domain. The randomness is then contained in the coefficients of the new boundary condition. This thin coating can be expressed by a random Ventcell boundary condition and yields a second order accurate solution in the scale parameter ε of the layer's thickness. With the help of the Karhunen-Loève expansion, we transform this random boundary value problem into a deterministic, parametric one with a possibly high-dimensional parameter $\mathbf{y}$. Based on the decay of the random fluctuations of the layer's thickness, we prove rates of decay of the derivatives of the random solution with respect to this parameter $\mathbf{y}$ which are robust in the scale parameter ε. Numerical results validate our theoretical findings.

Keywords: Thin layer equation, boundary value problem, random domain.

1. Introduction

Many practical problems in engineering lead to boundary value problems for an unknown function. In this article, we consider uncertainties in the geometric definition of the domain motivated by tolerances in the manufacturing processes or in a damaged boundary during the life of a mechanical device. Manufactured or damaged devices are close to a nominal geometry but differ of course from its mathematical definition. Since we are motivated by tolerances, we can make the crucial assumption that the random perturbations are small. By identifying domains with their boundary, domains close to the nominal domain D can be seen as a perturbation in the normal direction of the nominal boundary ∂D.

The most common approach to study boundary value problems with stochastic inputs is the Monte-Carlo method, see e.g. [5, 15, 25] and the references therein. In many situations,

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