



Extension of fuzzy TOPSIS method based on interval-valued fuzzy sets

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ABSTRACT

Decision making is one of the most complex administrative processes in management. In circumstances where the members of the decision making team are uncertain in determining and defining the decision making criteria, fuzzy theory provides a proper tool to encounter with such uncertainties. However, if decision makers cannot reach an agreement on the method of defining linguistic variables based on the fuzzy sets, the interval-valued fuzzy set theory can provide a more accurate modeling. In this paper the interval-valued fuzzy TOPSIS method is presented aiming at solving MCDM problems in which the weights of criteria are unequal, using interval-valued fuzzy sets concepts.

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1. Introduction

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is a popular approach to multiple criteria decision making (MCDM) problems that was proposed by Hwang and Yoon [1,2]. This method has been widely used in the literature (Abo-sinna and Amer [3], Agrawal et al. [4], Chen and Tzeng [5]). TOPSIS is a multiple criteria method to identify solution from a set of finite alternatives. The basic principle is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. In the TOPSIS, the performance ratings and the weights of the criteria are given as crisp values.

Among many cases, crisp data are inadequate to model real life situations. Chen [6] extends the TOPSIS method to fuzzy group decision making situations by considering triangular fuzzy numbers and defining crisp Euclidean distance between two fuzzy numbers. Tsaur et al. [7] convert fuzzy MCDM problem into a crisp one via centroid defuzzification and then solve the non-fuzzy MCDM problem using the TOPSIS method. Chu and Lin [8] changed fuzzy MCDM problem into a crisp one. Differing from the others, they first derive the membership functions of all the weighted ratings in a weighted normalized decision matrix and then by defuzzifying, convert them to crisp values. Triantaphyllou and Lin [9] develop a fuzzy version of TOPSIS method based on fuzzy

arithmetic operations, which leads to a fuzzy relative closeness for each alternative.

Except for Wang and Elhag [10], our survey shows that the vital shortcoming in the other mentioned methods are the loss of information (defuzzification) in initial steps of their procedure. Wang and Elhag's fuzzy TOPSIS method is based on alpha level sets and the fuzzy extension principle, which compute the fuzzy relative closeness of each alternative by solving the Non-linear programming models. Final ranking is obtained by defuzzifying the fuzzy relative closeness values.

In this paper we develop an interval-valued fuzzy TOPSIS (IVF-TOPSIS) to solve MCDM problems in which the performance rating values as well as the weights of criteria are linguistics terms which can be expressed in interval-valued fuzzy (IVFN) numbers. The remaining of this paper is organized as follows. In the next section, we will briefly introduce the TOPSIS method. Section 3 illustrates interval-valued fuzzy sets (IVFS). Section 4 describes developed TOPSIS method to solve interval-valued fuzzy MCDM problems. Section 5 investigates a numerical example including an application to select a manager for R&D department in a company. The paper is concluded in Section 6.

2. TOPSIS method

Yoon and Hwang [1,2], introduced the TOPSIS method based on the idea that the best alternative should have the shortest distance from an ideal solution. They assumed that if each attributes takes monotonically increasing or decreasing variation, then it is easy to define an ideal solution. Such a solution is composed of all the best attributes values achievable, while the worst solution is composed of all worst attribute values achievable.

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Suppose a multi criteria decision making problem having n alternatives, A_1, A_2, \dots, A_n and m criteria, C_1, C_2, \dots, C_m . Each alternative is evaluated with respect to the m criteria. All the values/ratings are assigned to alternatives with respect to decision matrix denoted by $X(x_{ij})_{n \times m}$. Let $W = (w_1, w_2, \dots, w_m)$ be the weight vector of criteria, satisfying $\sum_{j=1}^m w_j = 1$.

The TOPSIS method consists of the following steps:

- i. Normalize the decision matrix: the normalization of the decision matrix is done using the following transformation for each r_{ij} .

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^n x_{kj}^2}}, \quad i = 1, \dots, n; \quad j = 1, \dots, m. \quad (1)$$

Multiply the columns of the normalized decision matrix by the associated weights. The weighted and normalized decision matrix is obtained as:

$$V_{ij} = w_j \times r_{ij}; \quad i = 1, \dots, n \quad j = 1, \dots, m \quad (2)$$

where w_j represents the weight of the j th criterion.

- ii. Determine the ideal and negative ideal alternatives: the ideal and negative ideal alternatives are determined, respectively, as follows:

$$A^+ = \{v_1^+, v_2^+, \dots, v_m^+\} \\ = \left\{ \left(\max_j v_{ij} | j \in \Omega_b \right), \left(\min_j v_{ij} | j \in \Omega_c \right) \right\} \quad (3)$$

$$A^- = \{v_1^-, v_2^-, \dots, v_m^-\} \\ = \left\{ \left(\min_j v_{ij} | j \in \Omega_b \right), \left(\max_j v_{ij} | j \in \Omega_c \right) \right\} \quad (4)$$

where Ω_b is the set of benefit criteria and Ω_c is the set of cost criteria.

- iii. Obtain the distance of the existing alternatives from ideal and negative ideal alternatives: the two Euclidean distances for each alternative are, respectively, calculated as:

$$S_i^+ = \sqrt{\sum_{j=1}^m (v_{ij} - v_j^+)^2}, \quad i = 1, \dots, n \quad (5)$$

$$S_i^- = \sqrt{\sum_{j=1}^m (v_{ij} - v_j^-)^2}, \quad i = 1, \dots, n. \quad (6)$$

- iv. Calculate the relative closeness to the ideal alternatives: the relative closeness to the ideal alternatives can be defined as:

$$RC_i = \frac{S_i^-}{S_i^+ + S_i^-}, \quad i = 1, 2, \dots, m, \quad 0 \leq RC_i \leq 1. \quad (7)$$

- v. Rank the alternatives: according to the relative closeness to the ideal alternatives, the bigger is the RC_i , the better is the alternative A_i .

3. Interval-valued fuzzy sets

In fuzzy sets theory, it is often difficult for an expert to exactly quantify his or her opinion as a number in interval $[0, 1]$. Therefore, it is more suitable to represent this degree of certainty by an interval. Sambuc [12] and Grattan [13] noted that the presentation of a linguistic expression in the form of fuzzy sets is not enough. Interval-valued fuzzy sets were suggested for the first time by Gorzlczy [14] and Turksen [15]. Also Cornelis et al. [16] and Karnik and Mendel [17] noted that the main reason for proposing this new concept is the fact that in the linguistic modeling of a phenomenon, the presentation of the linguistic expression in the form of ordinary fuzzy sets is not clear enough. Wnag and Li [18] defined interval-

valued fuzzy numbers (IVFN) and gave their extended operations. Interval-valued fuzzy sets have been widely used in real-world applications. For instance, Kohout and Bandler [19] in a CLINAID system, Sambuc [12] in thyrodian pathology, Gorzlczy [14] and Bustine [20] in approximate reasoning, Turksen [21,22] in interval-valued logic and in preference modeling [15]. Based on definition of interval-valued fuzzy set in [14], an interval-valued fuzzy set A defined on $(-\infty, +\infty)$ is given by:

$$A = \{ (x, [\mu_A^L(x), \mu_A^U(x)]) \} \\ \mu_A^L, \mu_A^U : X \rightarrow [0, 1] \quad \forall x \in X, \quad \mu_A^L \leq \mu_A^U \\ \tilde{\mu}_A(x) = [\mu_A^L(x), \mu_A^U(x)] \\ A = \{ (x, \tilde{\mu}_A(x)) \}, x \in (-\infty, \infty) \quad (8)$$

where $\mu_A^L(x)$ is the lower limit of degree of membership and $\mu_A^U(x)$ is the upper limit of degree of membership.

Fig. 1 illustrates the membership value at x' of interval-valued fuzzy set A . Thereby, the minimum and maximum membership value of x' are $\mu_A^L(x')$ and $\mu_A^U(x')$, respectively.

Given two interval-valued fuzzy numbers $N_x = [N_x^-, N_x^+]$ and $M_y = [M_y^-, M_y^+]$, according to [23,24], we have:

Definition 1. If $\cdot \in (+, -, \times, \div)$, then $N \cdot M(x \cdot y) = [N_x^- \cdot M_y^-, N_x^+ \cdot M_y^+]$

Definition 2. The Normalized Euclidean distance between \tilde{N} and \tilde{M} is as follows:

$$D(\tilde{N}, \tilde{M}) = \sqrt{\frac{1}{6} \sum_{i=1}^3 [(N_{x_i}^- - M_{y_i}^-)^2 + (N_{x_i}^+ - M_{y_i}^+)^2]}$$

4. The proposed interval-valued fuzzy TOPSIS

In fuzzy MCDM problems, performance rating values and relative weights are usually characterized by fuzzy numbers. A fuzzy number is a convex fuzzy set, defined by a given interval of real numbers, each with a membership value between 0 and 1. Considering the fact that, in some cases, determining precisely of this value is difficult, the membership value can be expressed as an interval, consisting real numbers. In this paper criteria values' as well as criteria weights', are considered as linguistic variables. The concept of linguistic variable is very useful in dealing with situations that are too complex or ill-defined to be reasonably described in conventional quantitative expressions [11]. These linguistic variables can be converted to triangular interval-valued fuzzy numbers as depicted in Tables 1 and 2.

Let $\tilde{X} = [\tilde{x}_{ij}]_{n \times m}$ be a fuzzy decision matrix for a multi criteria decision making problem in which A_1, A_2, \dots, A_n are n possible alternatives and C_1, C_2, \dots, C_m are m criteria. So the performance of alternative A_i with respect to criterion C_j is denoted as \tilde{x}_{ij} . As illustrated in Fig. 2, \tilde{x}_{ij} and \tilde{w}_j are expressed in triangular interval-valued fuzzy numbers.

$$\tilde{x} = \begin{cases} (x_1, x_2, x_3) \\ (x'_1, x_2, x'_3) \end{cases}$$

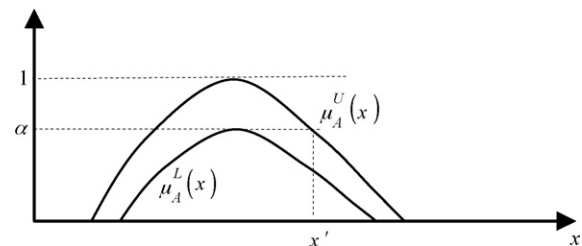


Fig. 1. Interval-valued fuzzy set.

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