Journal of Computational Physics 336 (2017) 1–35

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Non-deteriorating time domain numerical algorithms for Maxwell's electrodynamics ☆

S. Petropavlovsky [a,b], S. Tsynkov [b,c,*]

[a] National Research University Higher School of Economics, Moscow 101000, Russia
[b] Department of Mathematics, North Carolina State University, Box 8205, Raleigh, NC 27695, USA
[c] Moscow Institute of Physics and Technology, Dolgoprudny 141700, Russia

ARTICLE INFO

Article history:
Received 2 September 2016
Received in revised form 27 January 2017
Accepted 29 January 2017
Available online 2 February 2017

Keywords:
Unsteady electromagnetic waves
Maxwell's equations
Unbounded regions
Grid truncation
Artificial outer boundaries
Artificial boundary conditions (ABCs)
Non-reflecting boundaries
Perfectly matched layer (PML)
Long time deterioration
Loss of accuracy, loss of stability, error build-up
The Huygens' principle
Aft fronts of the waves
Lacunae of the solutions
Quasi-lacunae
Accumulation of charge
Guaranteed accuracy
Temporally uniform error bounds

ABSTRACT

The Huygens' principle and lacunae can help construct efficient far-field closures for the numerical simulation of unsteady waves propagating over unbounded regions. Those closures can be either standalone or combined with other techniques for the treatment of artificial outer boundaries. A standalone lacunae-based closure can be thought of as a special artificial boundary condition (ABC) that is provably free from any error associated with the domain truncation. If combined with a different type of ABC or a perfectly matched layer (PML), a lacunae-based approach can help remove any long-time deterioration (e.g., instability) that arises at the outer boundary regardless of why it occurs in the first place.

A specific difficulty associated with Maxwell's equations of electromagnetism is that in general their solutions do not have classical lacunae and rather have quasi-lacunae. Unlike in the classical case, the field inside the quasi-lacunae is not zero; instead, there is an electrostatic solution driven by the electric charges that accumulate over time. In our previous work [23], we have shown that quasi-lacunae can also be used for building the far-field closures. However, for achieving a provably non-deteriorating performance over arbitrarily long time intervals, the accumulated charges need to be known ahead of time.

The main contribution of the current paper is that we remove this limitation and modify the algorithm in such a way that one can rather avoid the accumulation of charge all together. Accordingly, the field inside the quasi-lacunae becomes equal to zero, which facilitates obtaining the temporally uniform error estimates as in the case of classical lacunae. The performance of the modified algorithm is corroborated by a series of numerical simulations. The range of problems that the new method can address includes important combined formulations, for which the interior subproblem may be non-Huygens', and only the exterior subproblem, i.e., the far field, is Huygens'.

© 2017 Elsevier Inc. All rights reserved.

☆ Work supported by the US Army Research Office (ARO) under grants # W911NF-14-C-0161 and # W911NF-16-1-0115, and by the US–Israel Binational Science Foundation (BSF) under grant # 2014048.

* Corresponding author at: North Carolina State University, Box 8205, Raleigh, NC 27695, USA.
E-mail addresses: spetrop@ncsu.edu (S. Petropavlovsky), tsynkov@math.ncsu.edu (S. Tsynkov).
URL: http://www4.ncsu.edu/~stsynkov/ (S. Tsynkov).

http://dx.doi.org/10.1016/j.jcp.2017.01.068
0021-9991/© 2017 Elsevier Inc. All rights reserved.

1. Introduction

In this paper, we develop and test a new algorithm that can furnish an exact artificial boundary condition (ABC) for the numerical simulation of electromagnetic waves propagating over unbounded regions, and can also stabilize any existing ABC or PML (perfectly matched layer) on arbitrarily long time intervals. The algorithm applies to a broad range of problems on $\mathbb{R}^3$ that may involve some sophisticated physical phenomena in the near field (generation, absorption, reflection, scattering, etc. of waves inside a bounded region) yet reduce to the propagation of electromagnetic waves in vacuum in the far field. In practice, only the near-field solution is actually computed, while the far field is truncated off and replaced by an ABC or a PML at the artificial outer boundary. The overall problem on $\mathbb{R}^3$ must be uniquely solvable and well-posed.

It is well known that many existing ABCs/PMLs are prone to error growth/instabilities in long-time simulations [1–18]. These undesirable phenomena may be due to continuous ill-posedness or weak well-posedness, discrete instability of a well-posed continuous problem, geometric issues such as near-edge or near-corner implementation of an ABC/PML, etc. Often, specific reasons for the deterioration are not known. This warrants the development of a general approach that can improve the long-time behavior of various methods.

A key advantage of our proposed algorithm is precisely its universality. It treats any given ABC or PML in the same manner and eliminates the need of finding out what actually causes the long-time deterioration in every particular case. The only limitation is that the chosen ABC or PML must be able to maintain its acceptable stable performance over reasonably long periods of time.[1] Those intervals of table performance can still be much shorter than the overall simulation time. An estimate (lower bound) for the interval of stable performance is needed for choosing the parameters of our algorithm. Other than that, its construction does not require any knowledge of why the performance of the original ABC/PML may deteriorate as the time elapses.

The algorithm is based on the Huygens' principle, or, in other words, it exploits the diffusionless propagation of waves in $\mathbb{R}^3$. Specifically, we use the following property of Maxwell's equations in vacuum: provided that the currents driving the pure Maxwell system are compactly supported in both space and time, the propagating electromagnetic waves have sharp aft (i.e., trailing) fronts, and the solution behind those is electrostatic, i.e., it does not evolve in time any longer. In our work [19], we called the region of space–time occupied by the steady-state solution *a quasi-lacuna* of Maxwell's equations. It generalizes the notion of classical lacunae, for which the solution behind the aft fronts is equal to zero, see [20] and also [21,22]. Once the outgoing waves have left the bounded computational domain, or, equivalently, once the latter entirely falls into the quasi-lacuna, there is no need to further update the fields and the simulation can stop. Hence, if the performance of the given ABC or PML during the corresponding finite time interval is acceptable, no subsequent adverse effects due this ABC or PML will ever arise.

In our work [23], we applied the lacunae-based algorithm to solving Maxwell's equations driven by currents that are compactly supported in space yet operate continuously in time (e.g., radiation of electromagnetic waves by an antenna with a given feed). The key idea is to partition the problem in time into a sequence of individual subproblems. Each of the latter appears driven by the currents that are compactly supported in both space and time. This facilitates the application of quasi-lacunae so that the solution of each subproblem does not need to be advanced in time beyond the point where it reaches the steady state on the domain of interest. Then, the overall solution is assembled by linear superposition. For every given moment of time, there are both steady-state and unsteady terms in the superposition sum that yields the solution. The "lifespan" of each unsteady term on the domain of interest is finite and non-increasing, because the domain size is finite and the propagation speed is finite. Moreover, the number of unsteady terms in the sum may never exceed a certain constant for all times, because the waves that leave the domain no longer need to be taken into account. On the other hand, the number of steady-state terms increases as the time elapses, because the electrostatic components of the solution stay on the computational domain even after it falls into the quasi-lacuna, i.e., after the unsteady waves leave. However, as the currents and charges that drive the Maxwell equations are given, the sum of all electrostatic contributions can be replaced by the electrostatic field at the final moment of time, which needs to be computed only once. Altogether, this yields a temporally uniform error estimate, which we proved in [23] and also corroborated with a series of 3D numerical experiments.

To extend the algorithm further, beyond solving the constant coefficient Maxwell equations driven by known currents, we still use the quasi-lacunae as a core integration tool but supplement them by some additional constructs that make the class of admissible formulations even wider:

(A) The original problem is decomposed into the interior problem (IP) and auxiliary problem (AP). The interior problem corresponds to the aforementioned sophisticated physics in the near field. It is formulated on a bounded domain and may be non-Huygens'. The auxiliary problem is formulated for the pure Maxwell equations on the entire $\mathbb{R}^3$ and is driven by the currents that depend on the solution to the IP. The AP satisfies the Huygens' principle. Solution of the AP provides the required closure for the IP at its outer boundary. The task of setting an ABC/PML is rather switched from the IP to the AP.

[1] The proposed methodology will not work in the case of very rapidly developing instabilities due to the treatment of the artificial outer boundaries. For example, it is not likely to offer a "fix" for the classical GKS-type instabilities.

Download English Version:

https://daneshyari.com/en/article/4967805

Download Persian Version:

https://daneshyari.com/article/4967805

Daneshyari.com