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Iterative Brinkman penalization for simulation of impulsively started flow past a sphere and a circular disc

Henrik Juul Spietz [a], Mads Mølholm Hejlesen [a], Jens Honoré Walther [a,b,*]

[a] Department of Mechanical Engineering, Technical University of Denmark, Building 403, DK-2800 Kgs. Lyngby, Denmark

[b] Computational Science and Engineering Laboratory, ETH Zürich, Clasiusstrasse 33, CH-8092 Zürich, Switzerland

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ABSTRACT

We present a Brinkman penalization method for three-dimensional (3D) flows using particle vortex methods, improving the existing technique by means of an iterative process. We perform simulations to study the impulsively started flow past a sphere at Re = 1000 and normal to a circular disc at Re = 500. The simulation results obtained for the flow past a sphere are found in qualitative good agreement with previously published results obtained using respectively a 3D vortex penalization method and a 3D vortex method combined with an accurate boundary element method. From the results obtained for the flow normal to a circular disc it is found that the iterative method enables the use of a time step that is one order of magnitude larger than required by the standard non-iterative Brinkman penalization method.

1. Introduction

Conventionally, the analysis of fluid-structure interaction requires the use of body-fitted grids to enforce the no-slip boundary condition on computational points located at the solid-fluid interface. The solid boundary condition may also be enforced with uniform grids using immersed boundary methods [1,2] or fictitious domain methods which rely on a modification of the governing equations. These methods may be interesting since they avoid time consuming generation of high quality, non-orthogonal grids, that require non-trivial solution algorithms.

One immersed boundary method is the Brinkman penalization method [3–5]. The principle of the method is to model a fluid flow in a porous medium by adding volume forcing to the governing equations. The porous medium flow tends to the primary flow past a non-porous immersed body as the permeability is reduced to zero in the part of the flow occupied by the solid body.

In the field of vortex methods several numerical schemes are based on this method [6–13]. Hejlesen et al. [7] proposed an iterative variation of the explicit scheme for the simulation of two-dimensional (2D) fluid flow past solid obstacles with the Vortex–In–Cell (VIC) algorithm. This iterative method uses a split-step scheme that overcomes a drawback of the conventional non-iterative schemes for vorticity-velocity formulation of the Navier–Stokes equations. For the non-iterative schemes an accurate enforcement of the solid boundary condition restricts the time step size because the numerical formulation lacks the global coupling of the elliptic kinematic equation. In this paper we show that the method is extendable to 3D flows. The proposed method is applied to the impulsively started flow past a sphere and the results are compared to results

* Corresponding author at: Department of Mechanical Engineering, Technical University of Denmark, Building 403, DK-2800 Kgs. Lyngby, Denmark. Fax: + 45 4588 4325.

E-mail address: jhw@mek.dtu.dk (J.H. Walther).

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by Mimeau et al. [13] and Ploumhans et al. [14]. The work by Mimeau et al. [13] includes a study of the flow past a sphere using the non-iterative implicit penalization. The focus of their studies is on the space-developing simulations since the velocity-vorticity relation is solved using a periodic Poisson solver with a velocity correction step to account for outflow of vorticity in the stream wise direction. Results are obtained for $\mathrm{Re} = 300$ and $\mathrm{Re} = 1000$ in simulated times long enough for the flows to reach a steady state. In the present study, we focus on the impact of an iterative process on accelerated flows, hence our approach avoids potential artifacts due to outflow boundary conditions. However, this limits the time span of the simulation due to growing extent of domain with time. Similarly Ploumhans et al. [14] studied this flow at various Reynolds numbers using a particle vortex method combined with a boundary element method (BEM) and panel-vortex diffusion to enforce the no-slip condition at the fluid-solid interface. They did not use outflow boundary conditions, but a mapping to a non-uniform grid to reduce the computational costs. Moreover, we present results for the simulation of the impulsively started flow normal to a circular disc of finite thickness at $\mathrm{Re} = 500$.

2. Methodology

2.1. The Brinkman penalization method

We solve the incompressible Navier–Stokes equations in a domain (Ω) consisting of a solid region ($\mathcal{S} \in \Omega$) and a fluid region ($\mathcal{F} = \Omega \setminus \mathcal{S}$). We introduce the Brinkman term that penalizes the difference between the solid velocity ($\boldsymbol{u}_s$) and the fluid velocity ($\boldsymbol{u}$) within the solid body to be close to zero [3]. Here we consider the vorticity-velocity formulation of the Navier–Stokes equations

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{\omega} = (\boldsymbol{\omega} \cdot \nabla)\boldsymbol{u} + \nu \nabla^2 \boldsymbol{\omega} + \nabla \times [\lambda \chi (\boldsymbol{u}_s - \boldsymbol{u})], \tag{1}$$

where ν is the kinematic viscosity of the fluid and λ is a penalization parameter, which may be seen as the inverse permeability of the solid region. χ is the characteristic function of $\mathcal{S}$

$$\chi(\boldsymbol{x}) = \begin{cases} 1 & \text{for } \boldsymbol{x} \in \mathcal{S} \\ 0 & \text{for } \boldsymbol{x} \in \mathcal{F}. \end{cases} \tag{2}$$

We derive the kinematic relation between the dependent variables by applying a Helmholtz decomposition for the velocity ($\boldsymbol{u}$) by requiring the vector potential ($\boldsymbol{\psi}$) to be solenoidal

$$\boldsymbol{u} = \nabla \times \boldsymbol{\psi} - \nabla \phi, \quad \nabla \cdot \boldsymbol{\psi} = 0 \tag{3}$$

The vorticity may then be related to the velocity by a Poisson equation through the vector potential or the velocity directly

$$\nabla^2 \boldsymbol{\psi} = -\boldsymbol{\omega}, \tag{4}$$

$$\nabla^2 \boldsymbol{u} = -\nabla \times \boldsymbol{\omega}. \tag{5}$$

We note that the Brinkman penalization term may be expanded as

$$\nabla \times [\lambda \chi (\boldsymbol{u}_s - \boldsymbol{u})] = \lambda \nabla \chi \times (\boldsymbol{u}_s - \boldsymbol{u}) + \lambda \chi (\boldsymbol{\omega}_s - \boldsymbol{\omega}), \tag{6}$$

where $\boldsymbol{\omega}_s = \nabla \times \boldsymbol{u}_s$. The term causes a production of vorticity in Eq. (1) due to residual in the velocity field ($\boldsymbol{u}_s - \boldsymbol{u}$) and vorticity field ($\boldsymbol{\omega}_s - \boldsymbol{\omega}$). $\nabla \chi$ is a vector field with non-zero magnitude only at the fluid-solid interface and is orientated normal to the interface. Hence, this production term is zero where the residual velocity ($\boldsymbol{u}_s - \boldsymbol{u}$) is also normal to the interface. As a consequence, when the Brinkman term is approximated separately from the elliptic kinematic relation between the dependent variables, the enforcement of the solid boundary condition will be delayed in time. This short-coming is distinctive in accelerated flows past objects, whose geometries have the majority of the surface area normal to the flow direction. This effect has been illustrated by Hejlesen et al. [7] for the impulsively started flow normal to a flat plate and it is the motivation for considering the impulsively started flow normal to a circular disc in the present study.

2.2. Numerical implementation of the Brinkman penalization method in a re-meshed vortex method

We solve the modified vorticity transport equation (Eq. (1)) in a split-step algorithm

$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \nabla \times [\lambda \chi (\boldsymbol{u}_s - \boldsymbol{u})], \tag{7}$$

$$\frac{D\boldsymbol{\omega}}{Dt} = (\boldsymbol{\omega} \cdot \nabla)\boldsymbol{u} + \nu \nabla^2 \boldsymbol{\omega}. \tag{8}$$

Our principle for solving Eq. (8) is to use the VIC method, where $\boldsymbol{\omega}$ is discretized onto N_p discrete particles carrying vorticity ($\boldsymbol{\omega}_p$). The particle vorticity is interpolated to a uniform grid of spacing h using a third order accurate interpolation kernel (M_4') [15] as