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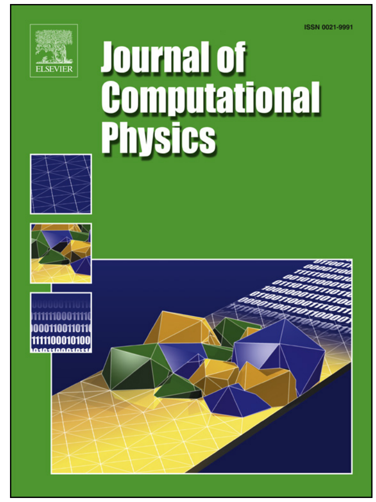

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Approximation of macroscopic conductivity for a multiscale model by using mortar methods

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Abstract

For a model with highly varying and multiscale conductivity, its macroscopic conductivity is approximated by using a mortar method. Macroscopic conductivity is useful in forming macroscopic models for porous media flow applications and in the setting of multiscale fast solvers. Many previous studies are based on the following procedure. Microscale models in each small cell are solved independently with an appropriate boundary condition and the solutions from the localized microscale problems are used to approximate the macroscopic conductivity. The size of the small cell and the boundary conditions affect the accuracy of the approximation. In this work, a mortar method is utilized to form localized microscale problems which are less sensitive to the boundary conditions. In addition, a simple and explicit formula for optimally determined macroscopic conductivity is derived by solving a nonlinear minimization problem. No postprocessing is thus required in our approach to calculate the macroscopic conductivity from the solutions of localized microscopic models. The new approach is numerically studied for various test models and compared to existing methods.

Keywords: Multiscale, Macroscopic, Microscopic, Mortar method

1. Introduction

Multiscale models are relevant in many practical applications such as flows through porous media and effective thermal conductivities for composite materials. Rather than resorting to a well-resolved fine scale model, upscaled macroscopic models are formulated and solved in practice. Previous studies for this direction have been mostly developed based on the homogenization theory, see [6, 8, 7, 21, 22, 4, 19]. We refer to [18, 14] for general reviews on upscaling or equivalent permeability.

In this work, a model elliptic problem with a multiscale conductivity is considered,

$$\begin{aligned} -\nabla \cdot (\sigma(x)\nabla u(x)) &= 0, \quad \forall x \in \Omega, \\ u(x) &= g_D(x), \quad \forall x \in \partial\Omega_D, \\ \frac{\partial u}{\partial n}(x) &= g_N(x), \quad \forall x \in \partial\Omega_N, \end{aligned} \tag{1}$$

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