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Multiscale gradient computation for flow in heterogeneous porous media

Rafael J. de Moraes [a,b,*], José R.P. Rodrigues [b], Hadi Hajibeygi [a], Jan Dirk Jansen [a]

[a] Department of Geoscience and Engineering, Faculty of Civil Engineering and Geosciences, Delft University of Technology, P.O. Box 5048, 2600, Netherlands

[b] Petrobras Research and Development Center – CENPES, Av. Horácio Macedo 950, Cidade Universitária, Rio de Janeiro, RJ 21941-915, Brazil

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ABSTRACT

An efficient multiscale (MS) gradient computation method for subsurface flow management and optimization is introduced. The general, algebraic framework allows for the calculation of gradients using both the Direct and Adjoint derivative methods. The framework also allows for the utilization of any MS formulation that can be algebraically expressed in terms of a restriction and a prolongation operator. This is achieved via an implicit differentiation formulation. The approach favors algorithms for multiplying the sensitivity matrix and its transpose with arbitrary vectors. This provides a flexible way of computing gradients in a form suitable for any given gradient-based optimization algorithm. No assumption w.r.t. the nature of the problem or specific optimization parameters is made. Therefore, the framework can be applied to any gradient-based study. In the implementation, extra partial derivative information required by the gradient computation is computed via automatic differentiation. A detailed utilization of the framework using the MS Finite Volume (MSFV) simulation technique is presented. Numerical experiments are performed to demonstrate the accuracy of the method compared to a fine-scale simulator. In addition, an asymptotic analysis is presented to provide an estimate of its computational complexity. The investigations show that the presented method casts an accurate and efficient MS gradient computation strategy that can be successfully utilized in next-generation reservoir management studies.

1. Introduction

Model-based reservoir management techniques typically rely on the information provided by derivatives. For instance, in sensitivity analysis studies, derivatives can be directly used to identify the most influential parameters in the reservoir response. Also, derivative information can be utilized in history matching [1] and control optimization [2] studies, where gradient-based optimization techniques are employed in the minimization/maximization of an objective function.

These types of model-based reservoir management studies are computationally demanding. They require multiple evaluations of the reservoir model in order to compute its response under the influence of different inputs. Reduced-order modeling (ROM) techniques have been employed to reduce the computational cost of the reservoir response evaluation. In

* Corresponding author at: Department of Geoscience and Engineering, Faculty of Civil Engineering and Geosciences, Delft University of Technology, P.O. Box 5048, 2600, Netherlands.

E-mail addresses: r.moraes@tudelft.nl (R.J. de Moraes), jrprodrigues@petrobras.com.br (J.R.P. Rodrigues), h.hajibeygi@tudelft.nl (H. Hajibeygi), j.d.jansen@tudelft.nl (J.D. Jansen).

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sensitivity analysis studies, response surface models and design of experiments are often used to reduce the computational costs (see, e.g., [3]). In history matching and optimization studies, techniques like upscaling [4], streamline simulation [5], and proper orthogonal decomposition [6] are employed to create reservoir models that are faster to evaluate. However, ROM and upscaling methods usually do not provide accurate enough system responses due to excessive simplifications of the fluid-rock physics and heterogeneous geological properties. To resolve this challenge, Multiscale (MS) methods have been developed [7–9].

MS methods solve a coarser simulation model, thus increasing the computational speed, while resolving the fine scale heterogeneities. Note that the specific multiscale methods addressed here, map between fine and coarse grids that are both at continuum (Darcy) scale, but with different computational grid resolutions. Moreover, the map between the nested fine and coarse grids is developed by using multiscale basis functions [7]. The basis functions are local solutions of the fine-scale equation, which are adaptively updated [10,11] and allow the MS coarse system to account for the fine-scale heterogeneities (which typically do not have separation of scale). Note that in contrast with MultiGrid (MG) methods, MS methods are not developed as linear solvers, but are most efficient if they are used – similar as in this paper – to provide approximate conservative fine-scale solutions (crucial for multiphase systems). MS methods are found efficient and accurate for large-scale reservoir models [12,13]. Important in this class of MS methods (compared to upscaling methods) is that the coarse-scale solutions are mapped onto the original fine scale, using the same basis functions. As such, errors can be calculated against the fine-scale reference systems (and not upscaled averaged ones). This allows for the development of convergent iterative MS procedures [14–16]. Recent developments include MS formulations for fractured media [17,18] with compositional effects [19,12,13] and complex well configurations [20] and gravitational effects [21]. In addition, algebraic formulation of the method has made it convenient to be integrated within existing simulators using structured [22,23] and unstructured [24–26] grids. The method has been also extended to fully-implicit formulations where all unknowns cross multiple dynamically-defined scales [27]. Although these developments are found efficient, they are mainly limited to forward simulation modeling. In this paper the focus is on the use of MS methods within reservoir management workflows.

Reservoir management techniques include optimization algorithms, in which calculation of derivatives plays an important role. The classical approaches for calculation of derivatives are either computationally expensive or inaccurate. For instance, numerical differentiation (see, e.g., [28,29]) suffers from discretization approximations and truncation errors, and is impractical when the number of parameters is large. Alternatively, analytical methods – Direct Methods (or Gradient Simulators) [30] and Adjoint Methods [31,32] – can provide accurate and efficient derivatives under appropriate conditions (to be further discussed in the Section 2). However, the use of analytical methods has not been extensively adopted mainly because they are code-intrusive and require a substantial implementation effort. On this issue, automatic differentiation can partly alleviate the burden of computing derivative information [33]. Additionally, most commercial simulators do not provide analytical derivative capabilities nor do they provide access to extend their functionality in this direction. Partially due to these drawbacks, ensemble methods such as the Ensemble Kalman Filter (EnKF) have become very popular in the data assimilation community [34]. Similarly, stochastic approximate gradient techniques such as ensemble optimization (EnOpt) and the stochastic simplex approximate gradient (StoSAG) method are increasingly being used for life cycle optimization [35,36]. These methods, however, by construction, provide an approximation of the gradient.

Multiscale gradient computation has been studied in the past. A Multiscale finite-volume Adjoint (MSADJ) method has been applied to sensitivity computation [37], where the global adjoint problem is solved via a set of coupled sub-grid problems described at a coarser scale. The coarse-scale sensitivities are then interpolated to the local fine grid by reconstructing the local variability of the model parameters with the aid of solving embedded adjoint sub-problems. In a follow up paper [38], the MSADJ method was efficiently applied to inverse problems of single-phase flow in heterogeneous porous media. Also, an efficient Multiscale Mixed Finite Element method has been developed for multiphase adjoint formulations, where both pressure and saturations are solved at the coarse scale [39]. In contrast to MSADJ, this method did not require fine-scale quantities.

The present development introduces a mathematical framework to compute sensitivities (gradients) in a multiscale strategy. The framework enables the same computational efficiency as existing multiscale methods [37–39]. However, its formulation allows for full flexibility with respect to the types of gradient information that are required by the different model-based reservoir management studies. This is achieved via an implicit differentiation strategy, as opposed to the more traditional Lagrange multiplier formulation. Also, the formulation naturally provides not only the Adjoint Method, but also the Direct Method. It is important to note that, although in this work the multiscale finite volume (MSFV) is being studied, the proposed MS-gradient method is not restricted to a specific MS method. Instead, it can be utilized in combination with any MS (and multi-level) strategy which is expressed in terms of restriction and prolongation operators.

This paper is structured as follows. First, the multiscale gradient computation method is derived based on the MS reservoir model equations and the respective model responses. The computation of the required prolongation (matrix of basis functions) operator derivatives is developed within the Multiscale Finite Volume (MSFV) formulation. Computational complexity of the method is also discussed from a theoretical point of view via asymptotic analysis of the algorithms. Thereafter, the Numerical Experiments section describes a systematic investigation of the validation, robustness, and accuracy of the MS-gradient method for test cases of increasing complexity. Because the proposed method is quite fundamental, the experiments are aimed at evaluating the gradient computation itself, rather than any specific application.