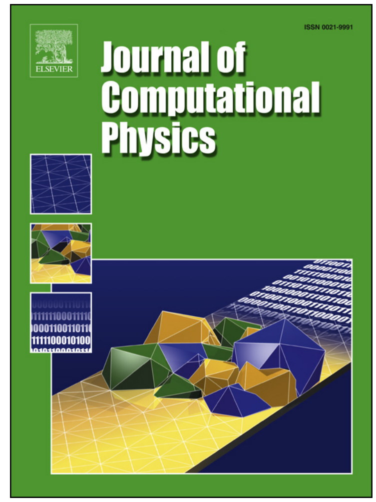
Accepted Manuscript



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J. Martín-Vaquero, B. Kleefeld

PII: S0021-9991(16)30399-0
DOI: http://dx.doi.org/10.1016/j.jcp.2016.08.042
Reference: YJCPH 6798

To appear in: *Journal of Computational Physics*

Received date: 2 June 2016
Revised date: 4 August 2016
Accepted date: 26 August 2016

Extrapolated Stabilized Explicit Runge-Kutta methods

J. Martín-Vaquero[a,*], B. Kleefeld[b]

[a]ETS Ingenieros industriales, Universidad de Salamanca. E37700, Bejar, Spain.
[b]Brandenburgische Technische Universität Cottbus, Institut für Mathematik, D-03044 Cottbus, Germany

Abstract

Extrapolated Stabilized Explicit Runge-Kutta methods (ESERK) are proposed to solve multi-dimensional nonlinear partial differential equations (PDEs). In such methods it is necessary to evaluate the function n_t times per step, but the stability region is $O(n_t^2)$. Hence, the computational cost is $O(n_t)$ times lower than for a traditional explicit algorithm. In that way stiff problems can be integrated by the use of simple explicit evaluations in which case implicit methods usually had to be used. Therefore, they are especially well-suited for the method of lines (MOL) discretizations of parabolic nonlinear multi-dimensional PDEs.

In this work, first $s-$stages first-order methods with extended stability along the negative real axis are obtained. They have slightly shorter stability regions than other traditional first-order stabilized explicit Runge-Kutta algorithms (also called Runge-Kutta-Chebyshev codes). Later, they are used to derive n_t-stages second- and fourth-order schemes using Richardson extrapolation. The stability regions of these fourth-order codes include the interval $[-0.01n_t^2, 0]$ (n_t being the number of total functions evaluations), which are shorter than stability regions of ROCK4 methods, for example. However, the new algorithms neither suffer from propagation of errors (as other Runge-Kutta-Chebyshev codes as ROCK4 or DUMKA) nor internal instabilities. Additionally, many other types of higher-order (and also lower-order) methods can be obtained easily in a similar way.

These methods also allow adaptation of the length step with no extra cost. Hence, the stability domain is adapted precisely to the spectrum of the problem at the current time of integration in an optimal way, i.e., with minimal number of additional stages. We compare the new techniques with other well-known algorithms with good results in very stiff diffusion or reaction-diffusion multi-dimensional nonlinear equations.

Keywords: Higher-order schemes, Multi-dimensional partial differential equations, Stabilized explicit Runge-Kutta methods, Variable-step length ODE solvers

*Corresponding author
Email addresses: jesmarva@usal.es (J. Martín-Vaquero), britta.kleefeld@b-tu.de (B. Kleefeld)