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Inflow/outflow with Dirichlet boundary conditions for pressure in ISPH

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In the present work we propose a new algorithm for open boundary treatment in ISPH. In the literature a few models for open boundary conditions are available, but most of them are applied to weakly compressible SPH (WCSPH) only. In our method the inflow/outflow is driven by true Dirichlet boundary conditions of the projected pressure field. We ensure the Dirichlet boundary condition by a particle mirroring technique at the open boundary to compute the pressure field. This procedure enables us to handle variable inlet velocities across the open boundary. The Dirichlet boundary conditions are introduced for the projected pressure matrix. We apply an error analysis for a Hagen–Poiseuille flow driven by a pressure gradient and demonstrate the robustness and accuracy with a flow around a cylinder and an oscillating flow, where inlet and outlet conditions periodically change. Additionally, a volume flux controller is presented in combination with variable pressure boundary conditions. Finally, the new open boundary treatment is applied to a bubble formation process during gas injection and validated with experimental results.

1. Introduction

Our goal is the use of the ISPH-method for simulations of multi-phase flow of immiscible fluids through solid structures like it happens in a gas injection nozzle in a bubble column reactor [1] or during the transport of fluids through porous media [2]. For these and similar examples robust treatment of open boundaries is crucial.

Hosseini and Feng [3] introduced a rotational pressure-correction scheme to deal with pressure boundary conditions (b.c.) in ISPH, but they still used a Neumann boundary condition when solving the pressure Poisson equation. Souto-Iglesias et al. [4] discussed Dirichlet and Neumann b.c. for pressure for the Lagrangian Moving Particle Semi-implicit Method (MPS) where the classical projection method of Chorin [5] was used. Recently Leroy et al. [6] introduced Dirichlet b.c. in ISPH, using unified semi-analytical wall boundaries [7].

Hirschler et al. [8] presented an approach for the ISPH-method using the classical SPH projection method of Cummins and Rudman [9]. In order to change the homogeneous Neumann boundary conditions for pressure at the open boundary to Dirichlet boundary conditions, they used mirrored particles at the inflow/outflow regions with a linear projection method. They split the open boundary into piece-wise defined mirror axes to minimize errors in the 0th moment of the kernel function. Regarding the Lagrangian movement of the particles all mirror axes are changing their position in each time-step. Hereafter we call this method the moving-mirror-axes method (MMA).

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In contrast to WCSPH pressure and density are treated separately in ISPH. Therefore, a limited error in the 0th moment of the kernel function can be accepted at the inflow/outflow region when using the corrected SPH formulation of Bonet and Lok [10]. This is not possible in standard WCSPH if a reasonable smoothing length of the kernel is applied [11].

In this work we used the same approach with mirrored particles from Hirschler et al. [8] to define a Dirichlet b.c. for pressure at the open boundary, but now we kept the position of the mirror axis constant. Additionally, we applied a divergence free condition for the velocity field at the open boundary, when solving the pressure Poisson equation. Together we call this the fixed-mirror-axes-method (FMA) later on. The combination of "corrected gradients of the corrected kernel" by Bonet and Lok [10] together with fixed mirror axes leads to an improved accuracy and robustness of the simulation at the open b.c.. The main advantage of the FMA method lays in the unique geometric specifications when and where new particles are added to or deleted from the system domain. Hence we can handle inflow and outflow over the same boundary segment straightforward since the flow profile just follows from the pressure boundary condition.

In contrast to boundary conditions with a fixed volume flux the Dirichlet b.c. for pressure can handle lateral variable flow profiles. This property is used in this work to introduce a volume flux controller, where the b.c. for pressure at the inlet is the controlled variable, to ensure a desired volume flux rate.

2. Model

Isothermal flow of incompressible Newtonian fluids in a continuous domain is described by the Navier–Stokes equations:

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho\, \mathbf{g} + \mathbf{F}_{wn}^{VOL} + \mathbf{F}_{wns}^{VOL}, \tag{1}$$

where the term $-\nabla p + \mu \nabla^2 \mathbf{u}$ results from a pressure gradient and viscous stress in the system and $\mathbf{g}$ represents any body force like gravity. $\mathbf{F}_{wn}^{VOL}$ and $\mathbf{F}_{wns}^{VOL}$ are volumetric forces which represent the forces acting at the fluid–fluid interface resulting from surface tension and the forces at the contact line between the two fluids and the wall, respectively. We call this model the CSF/CLF model where the Continuum Surface Force (CSF) was introduced by Brackbill et al [12] and brought to SPH by Morris [15], with

$$\mathbf{F}_{wn}^{VOL} = \mathbf{f}_{wn}\, \delta_{wn} = \sigma_{wn}\, \kappa_{wn}\, \mathbf{n}_{wn}, \tag{2}$$

where $\mathbf{f}_{wn}$, δ_{wn}, σ_{wn}, κ_{wn} and $\mathbf{n}_{wn}$ are the force per unit area, surface delta function, surface tension coefficient, curvature of the interface and the normal vector to the interface, respectively. An extension to the CSF model which includes forces at the contact line was proposed by Huber et al. [16]

$$\mathbf{F}_{wns}^{VOL} \cdot \nu_{ns} = (\sigma_{ns} - \sigma_{ws} + \sigma_{wn}\, \underbrace{(\hat{\nu}_{ns} \cdot \hat{\nu}_{wn})}_{-\cos\theta})\, \delta_{wns} \tag{3}$$

For incompressible fluids the continuity equation simplifies to

$$\frac{D\rho}{Dt} = -\rho\, (\nabla \cdot \mathbf{u}) = 0. \tag{4}$$

2.1. Implementation in SPH

We use the ISPH method introduced by Cummins and Rudman [9]. As kernel function we take the C2 spline function from Wendland [17]

$$W(\mathbf{r}, h) = \frac{7}{4\pi h^2} \begin{cases} (1 - \frac{q}{2})^4\, (2q + 1) & \text{if } q < 2, \\ 0 & \text{else}, \end{cases} \tag{5}$$

where $q = |\mathbf{r}|/h$ and the smoothing length is chosen with $h = 2.1$ in most cases because the CSF-model shows best results for this choice. For a single-phase Poiseuille flow it is additionally reduced to $h = 1.6$ to demonstrate the applicability of a reduced smoothing length in combination with the new open boundary treatment. The acceleration due to the viscous force is formulated according to Hu and Adams [18]

$$\left(\nu\, \nabla^2 \mathbf{u}\right)_i = \frac{1}{m_i} \sum_j \bar{\mu}_{ij} \left(V_i^2 + V_j^2\right) \frac{\mathbf{u}_{ij}}{r_{ij}} \frac{\partial \widetilde{W}_{ij}}{\partial r_{ij}}, \tag{6}$$

with $\widetilde{W}_{ij} = \widetilde{W}(\mathbf{r}_{ij}, h) = \widetilde{W}(\mathbf{r}_i - \mathbf{r}_j, h)$, $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$, $\mathbf{u}_{ij} = \mathbf{u}_i - \mathbf{u}_j$ and

$$\bar{\mu}_{ij} = \frac{2\mu_i \mu_j}{\mu_i + \mu_j}. \tag{7}$$