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Real-time solution of linear computational problems using databases of parametric reduced-order models with arbitrary underlying meshes

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ABSTRACT

A comprehensive approach for real-time computations using a database of parametric, linear, projection-based reduced-order models (ROMs) based on arbitrary underlying meshes is proposed. In the offline phase of this approach, the parameter space is sampled and linear ROMs defined by linear reduced operators are pre-computed at the sampled parameter points and stored. Then, these operators and associated ROMs are transformed into counterparts that satisfy a certain notion of consistency. In the online phase of this approach, a linear ROM is constructed in real-time at a queried but unsampled parameter point by interpolating the pre-computed linear reduced operators on matrix manifolds and therefore computing an interpolated linear ROM. The proposed overall model reduction framework is illustrated with two applications: a parametric inverse acoustic scattering problem associated with a mockup submarine, and a parametric flutter prediction problem associated with a wing-tank system. The second application is implemented on a mobile device, illustrating the capability of the proposed computational framework to operate in real-time.

1. Introduction

Many engineering applications require the ability to predict the behavior of physical systems in real-time. Among these, one can mention design optimization, optimal control, the solution of inverse problems, as well as uncertainty quantification. All of these applications typically incur a large number of numerical predictions for varying values of operating conditions. Usually, these are described by a set of parameters, and may define boundary conditions, initial conditions, and physical or shape parameters that in turn define the problem of interest and its underlying differential equations. Each of these predictions usually requires computationally intensive computations, as accurate discretizations of the underlying differential equations often lead to large-scale systems of equations.

Projection-based model order reduction (MOR) [1,2] reduces the large computational cost of a simulation performed using a high-dimensional model (HDM) by reducing the number of degrees of freedom in the computation. It achieves this objective by constructing first a reduced-order basis (ROB), then approximating the solution in the subspace described by

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this ROB. Currently, the most challenging problems faced by MOR are those associated with nonlinearities and parameter variations. For nonlinear systems, MOR requires addressing critical issues pertaining to solution discontinuities when these arise [3] (or regime changes in general), in order to achieve accuracy at low dimensionality. It also calls for an additional level of approximation to enable translating dimensionality reduction into large computational speedup [4–9]. Most importantly, the MOR of a parametric nonlinear system is also challenging due to the lack of robustness of a reduced-order model (ROM) with respect to parameter variations, which requires an appropriate training offline [10–13].

For parametric systems, MOR requires addressing important issues pertaining to the robustness with respect to parameter variations of both the construction process of a reduced-order model (ROM) and the performance of the resulting ROM. In this paper, the focus is set on addressing the latter issues in the context of linear systems. Specifically, the addressed challenge is that of designing a MOR reduction approach for a parametric linear system that is feasible, practical, and yet delivers a real-time capability for performing numerical predictions independently from the complexity of the associated application.

For parametric linear systems, ROM database approaches where reduced linear operators — and associated ROMs — of a common low dimension are pre-computed offline for sampled values of the parameters and stored have been recently developed [12–17]. In general, the idea is that the stored reduced linear operators can then be interpolated during an online phase of computations to construct in real-time linear ROMs at unsampled parameter values, and use these ROMs to perform real-time predictions on-the-fly.

Following the ROM database approach outlined above, special attention is paid in this paper to the interpolation step. To this effect, it is noted that important mathematical properties of reduced linear operators can be preserved when interpolation is performed on appropriate matrix manifolds [12,14,15,17]. Specifically, after appropriately mapping the linear reduced operators, interpolation can be carried out in the tangent space to a relevant matrix manifold. As long as the interpolation procedure preserves the tangent space, the interpolated quantity will also belong to the tangent space and can be mapped back to the manifold, leading to a properties-preserving interpolated reduced quantity. Nevertheless, the interpolation of local, linear, reduced operators is a challenge in itself since each linear reduced operator can be written in terms of a distinct set of generalized coordinates corresponding to the local ROBs associated with each local ROM. To address this issue, approaches based on congruent transformations were proposed in [14,16,17], for the case where the underlying HDMs arise from discretizations defined on a common mesh, which can be a rather severe limitation. Hence, these approaches are not applicable to the more frequent case where different HDMs are defined on different meshes. To remove this limitation, this paper introduces a novel approach that is also based on congruent transformations, but that addresses the challenge associated with arbitrary meshes. To this effect, its remainder is organized as follows.

The problem of interest and ROM database concept are formulated in Section 2. The consistency issue for reduced-order operators and its enforcement are discussed in Section 3 for both cases of common and arbitrary underlying meshes. The general approach for interpolating pre-computed consistent reduced operators in a database of linear ROMs is developed in Section 4. Special attention is paid to parameter sampling, data storage, and data exploitation. The proposed overall MOR approach is applied in Section 5 to the reduction of two parametric computational models. The first one is associated with a parametric inverse acoustic scattering problem featuring multiple meshes. The second model is associated with the flutter analysis on a mobile device of an aeroelastic system for flight conditions ranging from the subsonic to the supersonic regime. In both cases, it is shown that the proposed approach successfully enables real-time predictions. Finally, conclusions are given in Section 6.

2. Problem formulation and solution approach

In this paper, linear-time invariant parametric (LTIP) systems of one of the following two forms are considered:

1. First-order LTIP systems of the form

$$\mathbf{E}(\boldsymbol{\mu})\frac{d\mathbf{w}}{dt}(t) = \mathbf{A}(\boldsymbol{\mu})\mathbf{w}(t) + \mathbf{B}(\boldsymbol{\mu})\mathbf{u}(t) \tag{1}$$

or their formulation in the frequency domain

$$(j\omega\mathbf{E}(\boldsymbol{\mu}) - \mathbf{A}(\boldsymbol{\mu}))\mathbf{w}(\omega) = \mathbf{B}(\boldsymbol{\mu})\mathbf{u}(\omega),$$

where $\mathbf{E}$ and $\mathbf{A}$ are square high-dimensional matrices of dimension N acting on the high-dimensional state vector $\mathbf{w} \in \mathbb{R}^N$, $j^2 = -1$, $t \geq 0$ denotes time, and $\omega \geq 0$ frequency. The vector $\mathbf{u} \in \mathbb{R}^{N_i}$ denotes the input variable of dimension $N_i \ll N$ and $\mathbf{B} \in \mathbb{R}^{N \times N_i}$. All operators depend on a vector of $N_{\boldsymbol{\mu}}$ parameters $\boldsymbol{\mu} \in \mathcal{D} \subset \mathbb{R}^{N_{\boldsymbol{\mu}}}$.
For both formulations, an output quantity of interest (QoI) $\mathbf{y} \in \mathbb{R}^{N_o}$ is defined as

$$\mathbf{y} = \mathbf{G}(\boldsymbol{\mu})\mathbf{w} + \mathbf{H}(\boldsymbol{\mu})\mathbf{u}, \tag{2}$$

with $N_o \ll N$, $\mathbf{G} \in \mathbb{R}^{N_o \times N}$ and $\mathbf{H} \in \mathbb{R}^{N_o \times N_i}$.
Such first-order LTIP systems arise in the context of heat conduction, advection-diffusion and linearized computational fluid dynamics (CFD) applications, to name just a few.