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Rescaling of the Roe scheme in low Mach-number flow regions

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ABSTRACT

A rescaled matrix-valued dissipation is reformulated for the Roe scheme in low Mach-number flow regions from a well known family of local low-speed preconditioners popularized by Turkel. The rescaling is obtained explicitly by suppressing the pre-multiplication of the preconditioner with the time derivative and by deriving the full set of eigenspaces of the Roe–Turkel matrix dissipation. This formulation preserves the time consistency and does not require to reformulate the boundary conditions based on the characteristic theory. The dissipation matrix achieves by construction the proper scaling in low-speed flow regions and returns the original Roe scheme at the sonic line. We find that all eigenvalues are nonnegative in the subsonic regime. However, it becomes necessary to formulate a stringent stability condition to the explicit scheme in the low-speed flow regions based on the spectral radius of the rescaled matrix dissipation. With the large disparity of the eigenvalues in the dissipation matrix, this formulation raises a two-timescale problem for the acoustic waves, which is circumvented for a steady-state iterative procedure by the development of a robust implicit characteristic matrix time-stepping scheme. The behaviour of the modified eigenvalues in the incompressible limit and at the sonic line also suggests applying the entropy correction carefully, especially for complex non-linear flows.

1. Introduction

Low-speed preconditioning based on Chorin artificial compressibility has become widely used for computing low-speed flow configurations with numerical schemes developed for compressible flows. This local preconditioning technique was designed to achieve an optimal conditioning of the iterative procedure and to guaranty the proper scaling of the artificial dissipation when the Mach number approaches zero. The low-speed preconditioning approach has proved to be very efficient to overcome the accuracy issue of compressible flow solvers in the incompressible limit. Actually, the low-speed preconditioning should always be used since many industrial applications are characterized by mixed compressible and incompressible flows, over a wide range of Reynolds numbers.

However, this approach suffers from the complexity of its practical implementation. Since the local preconditioner modifies the characteristic relations, all boundary conditions based on characteristic variables or Riemann invariants must be reformulated accordingly. For large aerodynamics codes in which a large number of boundary conditions may be implemented, it is then necessary to reformulate most of the boundary conditions. Furthermore, the extension to unsteady flows is not trivial and without a special treatment, time-accuracy may be lost.

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Thus, a practical point of view has essentially motivated this contribution. For the compressible Euler equations, it was found interesting to investigate the effect of removing the pre-multiplication of the preconditioning matrix with the time-derivative of the independent flow variables. This formulation sometimes called improperly "preconditioning of the stabilization terms only" has been investigated particularly by Guillard and Viozat in [1,2] and by Birken and Meister in [3]. This formulation actually doesn't improve the conditioning and yields a large disparity in the eigenvalues of the matrix dissipation. However, this issue can be circumvented with some augmented Jacobi preconditioning, as mentioned in [4]. In addition, with this simplification, the explicit scheme recovers a basic structure with the centred scheme and stabilization terms. Then it becomes no longer necessary to reformulate the characteristic curves for the preconditioned Jacobian matrix and the time accuracy is preserved. Some recent attempts of improving the accuracy of conservative schemes in the low speed limit may have been also motivated by a drastic simplification of the implementation of the low-speed preconditioning. This is especially the case of the Rieper low Mach-number fix proposed in [5], and the Thornber et al. "Low Mach" LMRoe scheme [6] modifying jumps of the discrete velocities, which was further developed by Oßwald et al. with the "Low dissipation Low Mach" L^2Roe scheme [7]. This may also explain the success of the AUSM-family schemes and their modification for low Mach-number flows [8,9]. An alternative to modifying the matrix dissipation is to introduce a modified speed of sound, as considered by Rossow [10], and further extended in [11]. This approach was also investigated by Li and Gu in [12].

The problem of the accuracy in the asymptotic limit of the incompressible flow for the discretization of the normalized Euler equations was first addressed by Guillard and Viozat [1,2]. For their "preconditioning of the stabilization terms only" formulated with the Roe scheme as defined in [2], it was shown that the checkerboard modes for the leading and second-order pressure fields are cancelled out by the rescaled matrix dissipation and that the pressure field should be constant in space up to a fluctuation in space of order two. Furthermore, the authors have clearly pointed out a lack of dissipation of the standard conservative schemes in the incompressible limit.

A number of authors have further considered the discrete analysis based on the normalized equations for the asymptotic behaviour of the pressure and velocity fields. An "all-speed Roe scheme" has been developed by Li and Gu [13] in order to recover at the discrete level the divergence constraint of the leading order velocity and the Poisson equation for the second-order pressure, which are not satisfied by the preconditioned Roe scheme formulated in [1,2]. However, checkerboards modes are not automatically suppressed by their numerical flux and a low Mach number fix proposed by Rieper for the Roe's approximate Riemann solver [5] seems attractive, as combining the advantages in the incompressible limit of both approaches investigated in [1] and [13].

Over the last years, many modified Roe-type [1,5–7,13,14,12], AUSM-type [8,9], flux-splitting [15] or Godunov-type schemes [16,18] have been formulated to apply conservative finite-difference schemes to low-speed flows. It has been found necessary to propose a unified theoretical framework to analyse their respective discrete properties and to understand why they fail to be accurate in the incompressible limit without specific corrections. It is worth mentioning the work of Li and Gu for the analysis of Roe-type schemes [12], based on the flux splitting of the dissipation vector introduced in [17] and the contribution of Dellacherie for Godunov-type schemes [18] using the Hodge decomposition for solutions derived from the one-dimensional barotropic Euler equations.

On the other hand, few contributions have addressed the numerical stability of shock-capturing schemes adapted for low-speed flows. A Fourier Analysis is carried out by Dellacherie for the one-dimensional wave equations using the low Mach Godunov scheme and an explicit CFL condition is formulated for both the explicit and the implicit scheme [18]. For a formulation of the compressible Euler equations with "preconditioning of the stabilization terms only", the issue of the stability for a matrix-valued dissipation formulated from the Lax–Friedrich scheme is pointed out for the first time in [3] on the basis of the asymptotic behaviour of the largest eigenvalue in the incompressible limit. Formulating a stability criteria is also an essential feature when designing numerical schemes for complex flows, especially when an "all-speed scheme" is being developed. Results obtained by Birken and Meister clearly show for the Euler equations that the standard CFL condition used for the computation of compressible flows is no longer valid in the incompressible limit and that a stringent stability condition for the time step with $\Delta t \simeq \mathcal{O}(M^2)$ when the Mach number $M \to 0$ must be accounted for when the "preconditioning of the stabilization terms only" is considered. Nevertheless, the eigenspaces of the matrix-valued dissipation are not derived and a practical CFL condition for the local time step is not formulated explicitly for the fastest acoustic speed.

The main concern of this contribution is to reformulate a consistent matrix-valued dissipation with the low-speed limit and the transonic regime, and the corresponding stability condition, in the multidimensional case. We have considered the Roe scheme [19] as baseline formulation for the matrix dissipation. The necessary rescaling of the Roe scheme in the incompressible limit is formulated from a family of preconditioners popularized by Turkel [20–26]. The scheme is also sometimes termed as the Roe–Turkel scheme [2,18]. This reformulation corresponds to a drastic change of the stabilization terms and therefore the necessary Von Neumann criteria for the linear stability must be reconsidered completely. This can be achieved only by deriving the eigenvalues and the full set of the right and left eigenvectors of the matrix-valued dissipation, which surprisingly has never been done so far. The diagonalization of the rescaled matrix-valued dissipation must be achieved for the computation of complex flows, also because somehow an entropy fix may be used to prevent eigenvalues from approaching zero and to select the relevant physical solution satisfying the entropy condition across shocks.

With this reformulation, we lose the optimal conditioning but we save the essential feature of the accuracy in the incompressible limit. In particular, we see that by forcing the proper scaling of the dissipation matrix, we cannot avoid