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Coupling fluid–structure interaction with phase-field fracture

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ABSTRACT

In this work, a concept for coupling fluid–structure interaction with brittle fracture in elasticity is proposed. The fluid–structure interaction problem is modeled in terms of the arbitrary Lagrangian–Eulerian technique and couples the isothermal, incompressible Navier–Stokes equations with nonlinear elastodynamics using the Saint-Venant Kirchhoff solid model. The brittle fracture model is based on a phase-field approach for cracks in elasticity and pressurized elastic solids. In order to derive a common framework, the phase-field approach is re-formulated in Lagrangian coordinates to combine it with fluid–structure interaction. A crack irreversibility condition, that is mathematically characterized as an inequality constraint in time, is enforced with the help of an augmented Lagrangian iteration. The resulting problem is highly nonlinear and solved with a modified Newton method (e.g., error-oriented) that specifically allows for a temporary increase of the residuals. The proposed framework is substantiated with several numerical tests. In these examples, computational stability in space and time is shown for several goal functionals, which demonstrates reliability of numerical modeling and algorithmic techniques. But also current limitations such as the necessity of using solid damping are addressed.

1. Introduction

Both fluid–structure interaction (FSI) and fracture propagation are current but challenging topics with numerous applications in applied mathematics and engineering. For FSI literature we exemplary refer to the books [17,33,36,16,8] and for fracture mechanics we refer to [42,67,71,82,4,12,70]; and references cited therein are also emphasized. The goal of this work is to bring both frameworks together. This is of great interest since often FSI settings should also be able to account for fracture (or damage) of the solid part. On the other hand, single or multiple fractures or fracture networks can be found, for instance, in geomechanics, geophysics and porous media, which are possibly filled with fluids or coupled to surrounding flow. Thus, a framework that contains elastodynamics (which do also allow to account for large solid deformations), fluid flow, and a model for fracture representation and propagation is of current interest.

In classical FSI, the isothermal, incompressible Navier–Stokes equations are coupled with elastodynamics. The constitutive law in the solid is based on the geometrically nonlinear Saint-Venant Kirchhoff (STVK) model, see e.g., [20]. Here, three unknowns are sought: velocities, pressure and displacements. The FSI coupling technique is based on an interface-tracking method; namely the nowadays standard arbitrary Lagrangian–Eulerian (ALE) technique [25,47,50,63,32]. Here, the flow equations are re-written such that their coordinate system matches the Lagrangian framework of the solid. The resulting formulation using variational–monolithic coupling in the reference configuration is outlined in [49,68,74]. The key

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feature of the ALE approach is that the interface aligns with mesh edges and therefore interface-terms such as traction forces can be computed with high accuracy. In addition, it allows for many settings up to large solid deformations as long as the ALE mapping is a C^1-diffeomorphism.

On the other hand, brittle fracture propagation using variational techniques has attracted attention in recent years since the pioneering work in [35,11]. Since in FSI the constitutive stress tensor is generally nonlinear, we also refer to [21] who formulated quasistatic fracture growth using a variational setting employing nonlinear elasticity. The numerical approach [11] is based on Ambrosio–Tortorelli elliptic functionals [2,3]. Here, discontinuities in the displacement field across the lower-dimensional crack surface are approximated by an auxiliary function φ. This function can be viewed as an indicator function, which introduces a diffusive transition zone between the broken and the unbroken material. This zone has a half bandwidth ε, which is a model regularization parameter. From an application viewpoint, two situations are of interest for given fracture(s): first, observing the variation of the fracture width (crack opening displacement) and second, change of the fracture length. The latter situation is by far more complicated. However, both configurations are of importance and variational fracture techniques can be used for both of them.

Fracture evolutions satisfy a crack irreversibility constraint such that the resulting system can be characterized as a variational inequality. Our motivation for employing such a variational approach is that fracture nucleation, propagation, kinking, and curvilinear paths are automatically included in the model. In addition, explicit remeshing or reconstruction of the crack path is not necessary. The underlying equations are based on continuum mechanics principles that can be treated with (adaptive) Galerkin finite elements. On the contrary the underlying energy functional is not simultaneously convex in both solution variables [35] and a crucial difficulty. Another challenge is the resolution of ε in relation to the spatial discretization parameter $h < \varepsilon$, which requires local mesh adaptivity around the crack zone [43] when very small ε are of interest as well as a posteriori error estimation [5] and goal functional evaluations [79]; otherwise the computational cost becomes prohibitive.

An important modification of [35] towards a thermodynamically-consistent phase-field fracture (PFF) model has been accomplished in [59,56]. This approach has been extended to pressurized fractures in [61,62]. Here, the crack irreversibility constraint has been imposed through penalization. In phase-field fracture, two unknowns are sought: displacements and a phase-field function that determines the crack location. Recent advances and numerical studies towards pressurized and fluid-filled fracture and other multiphysics applications including thermo–elastic–plastic solids and coupling with a reservoir simulator have been considered in [62,60,73,58,55,80]. However, to the best of our knowledge coupling with classical FSI and the need to work with different coordinate systems has not yet been considered, which constitutes a major novelty of the present work.

As previously described, the solid part of FSI is based on elastodynamics, and therefore we accentuate the work of [9,52,13,51] who extended variational quasi-static brittle fracture to dynamic brittle fracture taking into account the solid acceleration term. Moreover, the authors of [52] introduced an elastic dissipation term, which is for wave propagation problems known as strong solid damping. This term improves the regularity of the solid velocity, see e.g., [37]. In the analysis for dynamic fracture this term was crucial, see [52, Remark 2.2], the corresponding theorems, and their conclusions. It turns out that in our numerical simulations we also need such a term; specifically for fractures that increase not only their width but also their length.

The goal in this paper is to couple ALE fluid–structure interaction with pressurized phase-field fracture. In order to achieve this task, we combine four models:

1. Nonlinear ALE fluid–structure interaction;
2. Crack representation and propagation in elastodynamics with phase-field;
3. Enforcing crack irreversibility via an augmented Lagrangian technique;
4. Pressurized phase-field fracture modeling in Lagrangian coordinates in a fixed reference domain.

In the first model, we deal with three types of nonlinearities: fluid convection, a geometrically nonlinear Green–Lagrange strain tensor, and finally the nonlinear ALE mapping. With regard to the second approach, we emphasize that we consider fixed fractures that only vary in their width as well as the more challenging configuration of propagating fractures. The third model does not need further comments and follows the ideas originally proposed in [34,40]. The fourth approach has been worked out for quasi-static fractures in porous media [61,62]. In the present work, this idea is extended to dynamic fractures in solid mechanics. The phase-field fracture equation is formulated in Lagrangian coordinates in order to match them with the ALE prescription of fluid–structure interaction.

The resulting formulation is consequently prescribed in a fixed, but arbitrary, reference domain and all coupling conditions are satisfied in a variational exact fashion on the continuous level. This formulation is now fully-coupled and can be written in terms of a Galerkin form. Then, numerical discretization is straightforward as the Rothe method (first time, then space) can be applied on the resulting semilinear form. These steps will be explained in great detail. The discretized, nonlinear coupled problem is solved with Newton's method with a modification that allows for an increase of the residual. This can be achieved either with an error-oriented line-search globalization [24] or simply using a classical residual-based approach that sometimes violates the convergence criterion.