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Waveform relaxation for the computational homogenization of multiscale magnetoquasistatic problems

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ABSTRACT

This paper proposes the application of the waveform relaxation method to the homogenization of multiscale magnetoquasistatic problems. In the monolithic heterogeneous multiscale method, the nonlinear macroscale problem is solved using the Newton–Raphson scheme. The resolution of many mesoscale problems per Gauß point allows to compute the homogenized constitutive law and its derivative by finite differences. In the proposed approach, the macroscale problem and the mesoscale problems are weakly coupled and solved separately using the finite element method on time intervals for several waveform relaxation iterations. The exchange of information between both problems is still carried out using the heterogeneous multiscale method. However, the partial derivatives can now be evaluated exactly by solving only one mesoscale problem per Gauß point.

1. Introduction

The recent use of the heterogeneous multiscale method (HMM [1]) in electrical engineering has allowed to accurately solve magnetoquasistatic (MQS) problems with multiscale materials, e.g. microstructured composites with ferromagnetic inclusions exhibiting hysteretic magnetic behavior [2,3]. The method requires the solution of one macroscale and mesoscale problems at each Gauß point of the macroscale problem (see Fig. 1) in a coupled formulation based on the Finite Element (FE) method. In [2,3] the coupled problem was monolithically time discretized by using equal step sizes at all scales and the resulting nonlinear problem was solved by an inexact parallel multilevel Newton–Raphson scheme. The finite-difference approach involves the resolution of 4 mesoscale problems in the three-dimensional case (respectively 3 mesoscale problems in two-dimensions) for computing the approximated Jacobian at each Gauß point.

The use of different time steps becomes important for problems involving different dynamics at both scales. In the case of the soft ferrite material studied in [4], for example, it was shown that capacitive effects occurring at the mesoscale could be accounted for by upscaling proper homogenized quantities in the macroscopic MQS formulation. Another relevant

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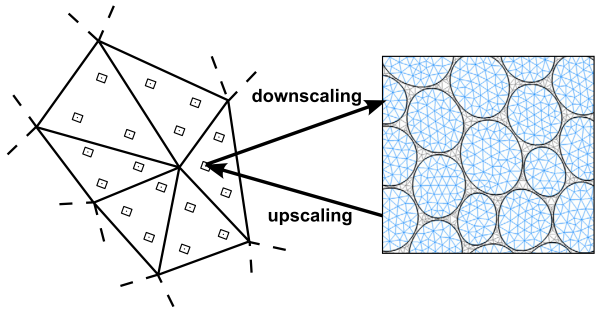

Fig. 1. Scale transitions between macroscale (left) and mesoscale (right) problems. Downscaling (Macro to meso): obtaining proper boundary conditions and the source terms for the mesoscale problem from the macroscale solution. Upscaling (meso to Macro): effective quantities for the macroscale problem calculated from the mesoscale solution [2,3].

case involves perfectly isolated laminations and soft magnetic composites (SMC) with eddy currents at the mesoscopic level (scales of the sheet/metallic grain) but without the resulting macroscopic eddy currents. The application of the HMM to problems involving such materials leads to a formulation featuring magnetodynamic problems at the mesoscopic scale and a magnetostatic problem at the macroscopic level. Thus, small time steps should be used at the mesoscale to resolve the eddy currents (especially with saturated hysteretic materials) while large time steps could be used to discretize the rather slowly-varying exciting source current at the macroscale level. Obviously, in such cases of different dynamics, the use of different time steps can help to reduce the overall computational cost.

In this paper we propose a novel approach that provides a natural setting for the use of different time steps. The approach applies the waveform relaxation method [5,6] to the homogenization of MQS problems: the macroscale problem and the mesoscale problems are solved separately on time intervals and their time-dependent solutions are exchanged in a fixed point iteration. The decoupling of the macroscale and the mesoscale problems and the independent resolution of these problems on time intervals has the potential to significantly reduce both the computation and communication cost of the multiscale scheme; in particular it allows to compute the Jacobian exactly at each Gauß point of the macroscale domain by solving only one mesoscale problem. As a drawback, waveform relaxation iterations are needed for the overall problem to converge in addition to the Newton–Raphson iterations on the meso- and macroscale. The latter exhibits quadratic convergence, while the fixed point iteration only leads to a linear convergence but is applied to waveforms instead of classical vector spaces. We present both approaches and compare the computational and the communication costs for both the monolithic and the waveform relaxation HMM.

The article is organized as follows: in Section 2 we introduce Maxwell's equations and the MQS problem. The weak form of the MQS problem is then derived using the modified vector potential formulation. Section 3 deals with the multiscale formulations of the HMM for the MQS problem along the lines of the works [2,3] with an emphasis on the coupling between the macroscale and the mesoscale problems. These formulations are valid for the monolithic and the waveform relaxation (WR) HMM. In Section 4 we develop a novel theoretical framework for the monolithic HMM. Using this framework we derive a reduced Jacobian from the Jacobian of the full problem using the Schur complement, similar to what has been proposed for the Variational Multiscale Method in [4]. Section 5 gives a short overview of the waveform relaxation method. The notion of weak and strong coupling are explained in the general context of coupled systems. The method is then used in Section 6 in combination with the HMM and gives rise to the newly developed WR-HMM. Section 7 is dedicated to the estimation of the computational cost for both the monolithic HMM and the WR-HMM. Formulae for the computation of costs for the monolithic HMM and the WR-HMM are derived and analyzed to give a hint on a possible reduction of the computational cost of both methods. Section 8 deals with an application case. We consider an application involving idealized soft magnetic materials (SMC) without global eddy currents. Convergence of the method as a function of the waveform relaxation iterations and the macroscale/mesoscale time stepping is numerically investigated.

2. The magnetoquasistatic problem

In an open, bounded domain $\Omega = \Omega_c \cup \Omega_c^C \subset \mathbb{R}^3$ (see Fig. 2) and $t \in \mathcal{I} = (t_0, t_{\mathrm{end}}] \subset \mathbb{R}$, the evolution of electromagnetic fields is governed by the following Maxwell's equations on $\Omega \times \mathcal{I}$, i.e.,

$$\operatorname{curl} \boldsymbol{h} = \boldsymbol{j} + \partial_t \boldsymbol{d}, \quad \operatorname{curl} \boldsymbol{e} = -\partial_t \boldsymbol{b}, \quad \operatorname{div} \boldsymbol{d} = \rho, \quad \operatorname{div} \boldsymbol{b} = 0,$$

and the constitutive laws, e.g. [7]

$$\boldsymbol{j}(\boldsymbol{x}, t) = \mathcal{J}\big(\boldsymbol{e}(\boldsymbol{x}, t), \boldsymbol{x}\big), \quad \boldsymbol{d}(\boldsymbol{x}, t) = \mathcal{D}\big(\boldsymbol{e}(\boldsymbol{x}, t), \boldsymbol{x}\big), \quad \boldsymbol{h}(\boldsymbol{x}, t) = \mathcal{H}\big(\boldsymbol{b}(\boldsymbol{x}, t), \boldsymbol{x}\big). \tag{2.1 a-c}$$

In these equations, $\boldsymbol{h}$ is the magnetic field [A/m], $\boldsymbol{b}$ the magnetic flux density [T], $\boldsymbol{e}$ the electric field [V/m], $\boldsymbol{d}$ the electric flux density [C/m^2], $\boldsymbol{j}$ the electric current density [A/m^2], and ρ the electric charge density [C/m^3]. The domain Ω_c contains conductors whereas the domain Ω_c^C contains insulators. Additionally, suitable initial conditions and boundary conditions must be imposed for the problem to be well posed.