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The HLLD Riemann solver based on magnetic field decomposition method for the numerical simulation of magneto-hydrodynamics

Xiaocheng Guo [a,*], Vladimir Florinski [b], Chi Wang [a]

[a] State Key Laboratory of Space Weather, National Space Science Center, Chinese Academy of Sciences, Beijing, 100190, China
[b] Department of Space Science, University of Alabama, Huntsville, AL, 35899, United States

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ABSTRACT

By splitting magnetic field into two components (internal plus external), we derived an extended formulation of the HLLD Riemann solver for numerical simulation of magneto-hydrodynamics (MHD). This new solver is backward compatible with the standard HLLD Riemann solver when the external component of the magnetic field is zero. Moreover, the solver is more robust than the standard HLLD solver in applications to low plasma β (the ratio between thermal and magnetic pressures) cases, where the thermal pressure may become negative from subtracting the kinetic and large magnetic energy from the large total energy density in a Godunov type numerical scheme. Our numerical tests show that the extended HLLD solver works well for the cases of magnetic field decomposition, and maintains high resolution similar to the standard HLLD.

1. Introduction

Magnetohydrodynamic (MHD) equations are widely used to describe the macro-scale behavior of plasma in many sub-disciplines of physics. Godunov type schemes are the commonly applied numerical methods for the Euler form of the conservative MHD equations [1,2]. In these schemes, zone-averaged MHD conserved values are updated from numerical fluxes at the grid interfaces at each time step. Riemann solvers are essential for the numerical flux calculation, and many kinds have been proposed in the literature. Some examples include the Roe-type solvers [3,2,4,5], the characteristic method [6], the local Lax–Friedrichs (LLF) method [7], and HLL-type solvers [8–10]. The LLF (also called Rusanov) solver is the simplest but also the most dissipative among those mentioned. The Roe-type solver includes every MHD waves modes, making it accurate for MHD simulation. However, this linearized solver does not preserve the positivity of density and pressure, and needs eigen-decomposition which is complicated and time-consuming in MHD. The characteristic method has properties similar to the Roe solver, but only applies to the Lagrangian form of MHD equations [11]. Comparing with the Roe-type, the HLL-type solvers are robust, positivity preserving and computationally inexpensive, which explains their popularity with many MHD applications. There are many extended versions of the HLL-type solver that differ in the choice of the middle states of the Riemann fan. The original HLL solver [9] includes a single middle state, the HLLC solver [12–14] has two, and the most advanced HLLD solver [15] has four. Theoretically, the single middle state HLL solver is too diffusive to re-

* Corresponding author.
E-mail address: guogxc@gmail.com (X. Guo).

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solve isolated contact discontinuities properly, so anti-diffusion terms are added to increase the accuracy in the so-called HLLEM solver [9]. However, a partial eigen-decomposition method is needed in the HLLEM solver, making it less efficient for MHD than for gasdynamic applications. The HLLD solver is the most accurate among the HLL-type solvers that exclude the eigen-decomposition because it has the most candidates for the numerical flux.

In space physics, the plasma β could be very low near the Sun or a planet, where strong internal magnetic field dominates in the inner region. For example, the plasma β could be less than 10^{-4} at $r \sim 4R_E$ (Earth radius), where r is the radial distance from the Earth's center. Such a low β will cause numerical issue for the conservative numerical scheme because the pressure may become negative because of subtracting the kinetic and the large magnetic energy from the large total energy density. Another problem may arise for the strong internal field. The magnetic energy density varies rapidly in space ($\sim 1/r^6$) if we treat the internal field as a dipole. In that case performing the spatial discretization of the magnetic energy density could produce unphysical oscillations and destroy the simulation. In order to overcome this difficulty, according to the previous experience [16], we need to decompose the magnetic field into two components, the static internal field and the time-dependent external field. As we will discuss in the next section, the spatial discretization is only applied to the external magnetic energy density, which avoids the numerical issue related to the total magnetic energy density. At the same time, the plasma β associated with the external field is much larger than that of the total field, and the above mentioned low β issue is avoided.

Based on this decomposition method for the magnetic field, the Roe-type Riemann solver is used straightforwardly for the decomposed MHD equation [16–18]. However, the HLLC and HLLD solvers must be modified if applied in a fully conservative decomposed MHD system [19]. An initial attempt has been reported for the HLLD solver in numerical simulations of the magnetosphere, but the method still needs further development [20]. In this paper, based on our former work [19], we will extend the HLLD solver for the decomposed MHD equation.

2. The decomposed MHD equations

The magnetic field $\mathbf{B}$ is split into two components, the internal field $\mathbf{B}_0$ and the external field $\mathbf{B}_1$,

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1, \tag{1}$$

where $\mathbf{B}_0$ is static and curl free $\nabla \times \mathbf{B}_0 = 0$, and $\mathbf{B}_1$ is time variable. The ideal MHD equations can be written in conservative form as [16]

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot F(\mathbf{U}) = 0, \tag{2}$$

where $\mathbf{U}$ is the vector of conservative variables, and $F(\mathbf{U})$ represents the corresponding flux vector,

$$\mathbf{U} = \begin{pmatrix} \rho \\ \rho\mathbf{u} \\ \mathbf{B}_1 \\ e_1 \end{pmatrix}, \quad F(\mathbf{U}) = \begin{pmatrix} \rho\mathbf{u} \\ \rho\mathbf{uu} + p\mathbf{I} - \mathbf{BB} - \frac{1}{2}B_0^2\mathbf{I} + \mathbf{B_0B_0} \\ \mathbf{uB} - \mathbf{Bu} \\ (e_1 + p_1)\mathbf{u} - (\mathbf{u}\cdot\mathbf{B_1})\mathbf{B}_1 + (\mathbf{B}_0 \times \mathbf{u}) \times \mathbf{B}_1 \end{pmatrix} \tag{3}$$

where ρ is density, and $\mathbf{u} = (u_x, u_y, u_z)$ is velocity. The total pressure p, p_1, and the total energy density e_1 are defined as

$$p = p_{gas} + \frac{B^2}{2}, \quad p_1 = p_{gas} + \frac{B_1^2}{2}, \quad e_1 = \frac{p_{gas}}{\gamma - 1} + \frac{\rho u^2}{2} + \frac{B_1^2}{2}, \tag{4}$$

where p_{gas} is the thermal pressure of plasma. In one dimension (the x direction) the system (3) reads

$$\mathbf{U} = \begin{pmatrix} \rho \\ \rho u_x \\ \rho u_y \\ \rho u_z \\ B_{1x} \\ B_{1y} \\ B_{1z} \\ e_1 \end{pmatrix}, \quad F(\mathbf{U}) = \begin{pmatrix} \rho u_x \\ \rho u_x^2 + p - B_x^2 - \frac{1}{2}B_0^2 + B_{0x}^2 \\ \rho u_x u_y - B_x B_y + B_{0x}B_{0y} \\ \rho u_x u_z - B_x B_z + B_{0x}B_{0z} \\ 0 \\ u_x B_y - u_y B_x \\ u_x B_z - u_z B_x \\ (e_1 + p_1)u_x - (\mathbf{u}\cdot\mathbf{B_1})B_{1x} + (\mathbf{B_1}\cdot\mathbf{B}_0)u_x - (\mathbf{B_1}\cdot\mathbf{u})B_{0x} \end{pmatrix}. \tag{5}$$

B_{1x} is constant along x direction because the corresponding flux is zero (divergence free constraint). In numerical applications, it is difficult to maintain the zero divergence for the magnetic field if no extra cleaning methods is applied [21,22,18].

The ideal MHD equations have seven eigenvalues λ_{1-7}, including one entropy wave, two Alfven waves, two fast and two slow magneto-acoustic waves:

$$\lambda_4 = u_x, \quad \lambda_{2,6} = u_x \mp c_a, \quad \lambda_{1,7} = u_x \mp c_f, \quad \lambda_{3,5} = u_x \mp c_s \tag{6}$$