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# High-order solution methods for grey discrete ordinates thermal radiative transfer

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ABSTRACT

This work presents a solution methodology for solving the grey radiative transfer equations that is both spatially and temporally more accurate than the canonical radiative transfer solution technique of linear discontinuous finite element discretization in space with implicit Euler integration in time. We solve the grey radiative transfer equations by fully converging the nonlinear temperature dependence of the material specific heat, material opacities, and Planck function. The grey radiative transfer equations are discretized in space using arbitrary-order self-lumping discontinuous finite elements and integrated in time with arbitrary-order diagonally implicit Runge–Kutta time integration techniques. Iterative convergence of the radiation equation is accelerated using a modified interior penalty diffusion operator to precondition the full discrete ordinates transport operator.

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## 1. Introduction

The goal of this work is to accurately solve the time-dependent thermal radiative transfer (TRT) equations. The TRT equations are a nonlinear system of equations that describe the conservation and transfer of energy between a photon (radiation) field and a stationary material. Solution of the TRT equations is an important component in several areas of physics including, but not limited to, astrophysics, inertial confinement fusion, and high energy density laboratory physics experiments.

It has previously been shown for linear neutron transport spatially discretized with discontinuous Galerkin finite elements (DFEM) that an increase of the trial space polynomial degree results in increased accuracy per unknown as compared to lower degree polynomial trial space DFEM solutions [1]. Additionally, as computing hardware advances, the number of floating point operations (FLOPs) available per memory operation continues to increase [2]. Computer architecture is not yet to the point where the higher number of FLOPs per unknown associated with higher order methods is completely masked by memory latency and relative narrowing of memory bandwidth [3–5]. However, we are quickly approaching an era in computing where increased accuracy through increased FLOPs per unknown will no longer result in a corresponding increase in time to solution; the main computing performance bottleneck will likely be memory latency or memory bandwidth.

In radiative transfer, material absorption opacities are strong functions of temperature; the common assumption being that opacities, $\sigma$, vary proportionally to $\frac{1}{T^3}$ [6–9]. Since temperature varies within each spatial zone, accurate TRT spatial

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discretization schemes must explicitly account for this within-zone opacity variation. As shown in [10] for neutron transport, the common zone-wise constant cross section approximation limits DFEM order of convergence to at most second order in space, regardless of trial space degree.

We use an overall solution strategy similar to that of Morel et al. [11] to solve the grey thermal radiative transfer equations: discrete ordinates to treat the angular dependence of the radiation equations, quasi-Newton iteration on material temperature, and approximation of the Planck function spatial dependence as a polynomial contained within the DFEM trial space. However, our work varies from that of [11] in several ways. First, in this work we use arbitrary-order self-lumping DFEM [10] rather than limiting our derivation to lumped linear DFEM. Likewise, we derive our equations using arbitrary-order, arbitrary stage count diagonally implicit Runge–Kutta (SDIRK) time integration [12], rather than considering only first order, single stage, implicit Euler time integration. Additionally, we fully converge the nonlinear temperature dependence of the grey TRT at every time integration stage, rather than performing a single quasi-Newton iteration. Each quasi-Newton iteration fully accounts for the nonlinear temperature dependence of the Planck function. Material property nonlinear dependencies are resolved by updating all temperature dependent material properties with every new temperature iterate. New temperature iterates come from the linearization of the Planck function temperature dependence. When the Planck linearization is fully converged, the nonlinear dependencies of the material properties are also converged. Finally, we do not assume that opacity and heat capacity are zone-wise constant; we explicitly account for the within zone variation of material properties via our self-lumping DFEM [10].

The remainder of this paper is divided as follows. In Section 2 we derive the discrete ordinate, grey thermal radiative transfer equations spatially discretized with arbitrary order DFEM and integrated in time with an arbitrary stage count SDIRK method. Additionally in Section 2, we apply the modified interior penalty diffusion synthetic operator [13,14] to accelerate the iterative convergence of our linearized arbitrary order DFEM radiation equation. An overview of our solution algorithm is given in Section 3. In Section 4 we present a series computational problems and results that: verify our implementation, demonstrate the asymptotic orders of convergence of our new methodology, and show that the proposed methods yield tangible benefits for physically meaning full simulations. In Section 4 we also briefly discuss the computational characteristics of our algorithm. Finally, we discuss our results and offer concluding remarks in Section 5.

## 2. Discretization of the thermal radiative transfer equations

We begin with the slab geometry, grey, thermal radiative transfer equations:

$$\frac{1}{c}\frac{\partial I}{\partial t} + \mu\frac{\partial I}{\partial x} + \sigma_t I = \frac{\sigma_s}{4\pi}\phi + \sigma_a B + Q_I \tag{1a}$$

$$C_v\frac{\partial T}{\partial t} = \sigma_a\left(\phi - 4\pi B\right) + Q_T\,, \tag{1b}$$

where $x$ is position in the slab; $t$ is time; $c$ is the speed of light; $I$ is the frequency integrated intensity, $I = I(x,\mu,t)$; $\mu$ is the directional cosine of the intensity relative to the $x$ axis; $T$ is the material temperature, $T = T(x,t)$; $\phi$ is the frequency and angle integrated intensity, $\phi = \phi(x,t)$; $\sigma_t$, $\sigma_s$, and $\sigma_a$ are respectively the total, scattering, and absorption opacities, with each opacity being a function of space and material temperature; $B$ is the Planck function, $B(T)$; $C_v$ is the material heat capacity, $C_v = C_v(x,T)$; $Q_I$ is a driving radiation source, $Q_I = Q_I(x,\mu,t)$; and $Q_T$ is a driving temperature source, $Q_T = Q_T(x,t)$. Driving sources are considered for generality and the use of the method of manufactured solutions. In Eq. (1a), we have assumed only isotropic scattering, and define the angle integrated intensity as:

$$\phi(x) = 2\pi\int_{-1}^{1} I(x,\mu)\,d\mu\,. \tag{2}$$

The frequency integrated Planck function, with Planck radiation constant, $a$, is:

$$B(T) = \frac{acT^4}{4\pi}\,. \tag{3}$$

Introducing the discrete ordinates approximation,

$$\phi \approx 2\pi\sum_{d=1}^{N_{dir}}\omega_d I_d\,, \tag{4}$$

where $\mu_d, \omega_d$ are direction and weight pairs of a given angular quadrature set with $\sum_{d}^{N_{dir}}\omega_d = 2$, $I_d = I(\mu_d)$, and $Q_{I_d} = Q_I(\mu_d)$, Eq. (1a) becomes:

$$\frac{1}{c}\frac{\partial I_d}{\partial t} + \mu_d\frac{\partial I_d}{\partial x} + \sigma_t I_d = \frac{\sigma_s}{4\pi}\phi + \sigma_a B + Q_{I_d}\,. \tag{5}$$