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A new framework for magnetohydrodynamic simulations with anisotropic pressure

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A B S T R A C T

We describe a new theoretical and numerical framework for magnetohydrodynamic (MHD) simulations with an incorporated anisotropic pressure tensor, which can play an important role in collisionless plasmas. The classical approach to handle the anisotropy is based on application of the double adiabatic approximation, assuming that the pressure tensor is well described only by those components that are oriented parallel and perpendicular to the local magnetic field. This gyrotropic assumption, however, fails around magnetically neutral regions, where the cyclotron period may become comparable to or even longer than the system's dynamical time, which causes a singularity in the mathematical expression. In this paper, we demonstrate that this singularity can be completely removed by direct use of the 2nd-moment of the Vlasov equation, combined with an ingenious gyrotropization model. Numerical tests are used to verify that our model properly reduces to the standard MHD results or the double adiabatic formulation in an asymptotic manner under the limit of fast isotropization and fast gyrotropization, respectively.

1. Introduction

Space and astrophysical phenomena often occur in collisionless plasmas, where the gas is so hot and dilute that the mean free path of charged particles becomes longer than the system's scale size. To investigate such complicated collisionless systems, numerical simulations can be powerful tools. In fact, particle-in-cell (PIC) and Vlasov simulations are typical numerical methods to solve the Vlasov–Maxwell system, which represents a set of fundamental equations describing the time evolution of the velocity distribution function and the electromagnetic fields. Although these models can capture all important kinetic physics self-consistently, because of limited computational resources it is still hard to apply these methods to phenomena occurring on scales that significantly exceed the kinetic scales, such as those associated with Larmor radii and/or inertial lengths (but which are still smaller than the relevant mean free paths). The Earth's magnetosphere (e.g., [23,12,34,4]) and the solar wind (e.g., [27,48,5]) are typical examples of such large-scale collisionless plasmas in the solar system. Using an astrophysical example, radiatively inefficient accretion flow models applied to accretion disks are also thought to represent collisionless plasmas (e.g., [31,39,40]).

One classical way of dealing with both dynamical and kinetic scales is the so-called kinetic magnetohydrodynamics (MHD) approach, which can take into account some kinetic effects. This philosophy has given rise to the well-known double adiabatic approximation, also known as the Chew–Goldberger–Low (CGL) model [7], which pays special attention to the

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effects of anisotropy in a distribution function. Given that the orbit of a charged particle in a magnetized plasma essentially consists of a combination of its the gyromotion around a magnetic field line and its parallel motion along the field line, the distributions of the kinetic energies contained in these independent motions may differ from each other. This situation requires us to extend the standard MHD model with a scalar pressure so as to handle the anisotropic pressure tensor. The double adiabatic approximation is a natural extension of the one-temperature MHD approach, where only the parallel and perpendicular components of the pressure tensor are solved. This is one of the simplest equations of state as a closure model of the moment hierarchy, assuming that the pressure is completely gyrotropic and the third- and higher-order moments are neglected. The properties of this formulation have been studied for decades and it has achieved a certain degree of success (e.g., [28,20,21,35]).

There have been several approaches to construct models beyond the double adiabatic approximation. One noteworthy effort retains the contribution of the higher-order moments by employing a sophisticated heat-flux model, such as Landau closure (e.g., [18,43]). A different approach was taken by Hada et al. [17]. It relaxes the assumption of the presence of a gyrotropic pressure tensor and introduces a new time scale to describe gyrotropization. This allows finite non-gyrotropy to remain. Wang et al. [47] attempted to combine these two approaches together. In addition, asymptotic-preserving (AP) schemes [33] may also shed new light on the treatment of anisotropic pressure in some extreme cases.

The gyrotropic formulation in the CGL approximation, however, involves numerical and theoretical difficulties in handling magnetic null points. This comes from the fact that the direction of the magnetic field must be defined for the decomposition of a pressure tensor into its parallel and perpendicular components. From a numerical point of view, determination of the unit vector parallel to the magnetic field, $\hat{\mathbf{b}} = \mathbf{B}/|B|$, will cause division by zero in a magnetic null point, which severely hampers numerical simulations. If one employs the form of a conservation law, the conservative variable pertaining to the first adiabatic invariant includes the magnetic field in the denominator as well. This drawback may become critical when, for example, considering a current sheet without a guide field, which contains a magnetically neutral line in its own right. The role of pressure anisotropy as pertaining to collisionless magnetic reconnection cannot be studied, therefore, in the framework of the CGL equations.

This breakdown apparently comes from the strong assumption that a pressure tensor can be well described by the gyrotropic form. In other words, the gyro-motion is well-defined on a much shorter time scale than the relevant dynamical time scale. If the magnetic field is so weak that the gyro period becomes comparable to the dynamical time scale, the parallel and perpendicular motions cannot be distinguished from each other and the gyrotropic approximation is no longer valid. As long as we are stuck with the gyrotropic limit, therefore, the problem of division by zero at magnetic null points will not be eliminated completely, regardless of the form of the equations of states employed.

With this in mind, we relax the assumption of gyrotropic pressure and extend the equations of state so as to allow finite deviations from the gyrotropic formulation. This paper focuses on such a natural extension of the MHD approach following the context of a governing equation that describes a more general form of the pressure tensor. Desirable characteristics of the newly defined theoretical and numerical framework are (1) avoidance of numerical difficulties owing to division by zero in magnetically neutral regions; (2) removal of any temporal and spatial scales related to kinetic physics; (3) convergence to the gyrotropic and isotropic formulations, respectively, under appropriate limits; and (4) a simple modification of an existing MHD code. In this paper, we successfully derive a new framework that satisfies these requirements by evolving a 2nd-rank pressure tensor directly, and we develop a corresponding, extended MHD code.

The present paper is organized as follows. First, we derive our analytical formulation in Section 2. Next, Section 3 describes the actual implementation of our simulation code based on the finite-difference approach. The numerical behavior is tested in Section 4. Finally, Section 5 contains a summary as well as our concluding remarks.

2. Formulation

2.1. Generalized energy conservation law

In this subsection, we will briefly derive our fluid model, starting from the Vlasov equation,

$$\frac{\partial f_s}{\partial t} + \mathbf{v}_s \cdot \nabla f_s + \frac{q_s}{m_s}\left(\mathbf{E} + \frac{\mathbf{v_s}}{c} \times \mathbf{B}\right) \cdot \nabla_{\mathbf{v}} f_s = 0, \tag{1}$$

where the subscript s indicates the species of charged particles—in this paper, ions, i, and electrons, e. The other notations are standard. Taking the second moment of the particle velocity $\mathbf{v}_s$ in Eq. (1), we obtain a kinetic stress tensor equation:

$$\begin{aligned}&\frac{\partial}{\partial t}\left(m_s n_s \mathbf{V}_s \mathbf{V}_s + \mathbf{P}_s\right) + \nabla \cdot \left[m_s n_s \mathbf{V}_s \mathbf{V}_s \mathbf{V}_s + \left(\mathbf{V}_s \mathbf{P}_s\right)^S + \mathbf{Q}_s\right] \\ &\quad = q_s n_s \left[\mathbf{V}_s\left(\mathbf{E} + \frac{\mathbf{V}_s}{c} \times \mathbf{B}\right)\right]^S + \frac{q_s}{m_s c}\left(\mathbf{P}_s \times \mathbf{B}\right)^S,\end{aligned} \tag{2}$$

where the superscript S denotes symmetrization. More specifically, $(\mathbf{VP})^S_{ijk} = V_i P_{jk} + V_j P_{ik} + V_k P_{ij}$ and $(\mathbf{VE})^S_{ij} = V_i E_j + V_j E_i$, respectively. The moment variables are defined as