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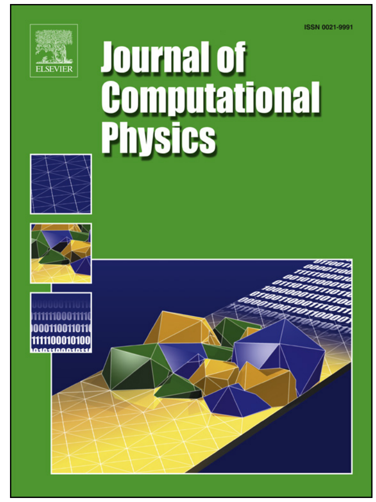

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Nicola Castelletto, Joshua A. White, Massimiliano Ferronato

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Scalable Algorithms for Three-Field Mixed Finite Element Coupled Poromechanics

Nicola Castelletto[a,*], Joshua A. White[b], Massimiliano Ferronato[c]

[a] *Energy Resources Engineering, Stanford University, United States*
[b] *Atmospheric, Earth and Energy Division, Lawrence Livermore National Laboratory, United States*
[c] *Department of Civil, Environmental and Architectural Engineering, University of Padova, Italy*

Abstract

We introduce a class of block preconditioners for accelerating the iterative solution of coupled poromechanics equations based on a three-field formulation. The use of a displacement/velocity/pressure mixed finite-element method combined with a first order backward difference formula for the approximation of time derivatives produces a sequence of linear systems with a 3×3 unsymmetric and indefinite block matrix. The preconditioners are obtained by approximating the two-level Schur complement with the aid of physically-based arguments that can be also generalized in a purely algebraic approach. A theoretical and experimental analysis is presented that provides evidence of the robustness, efficiency and scalability of the proposed algorithm. The performance is also assessed for a real-world challenging consolidation experiment of a shallow formation.

Keywords: poromechanics, preconditioners, iterative methods, mixed formulation, algebraic multigrid

1. Introduction

Coupling between fluid flow and mechanical deformation is a key factor in many subsurface engineering applications, such as hydrocarbon recovery [1, 2], subsurface hydrology [3–5], geothermal energy extraction [6, 7], and geologic carbon storage [8–10]. A similar behavior often governs porous media beyond geoscience systems, e.g. biomechanical modeling of bone or soft tissue deformations [11, 12]. The fundamental mathematical framework governing coupled fluid flow and deformation—generally referred to as *poroelasticity* [13]—was established by Biot [14]. Today, the poroelasticity theory and more sophisticated extensions, e.g. [15], have a well-established theoretical foundation. However, the accurate and efficient numerical simulation of tightly coupled poromechanical systems still poses severe computational challenges that require advanced discretization techniques and linear/non-linear solvers to obtain reliable modeling predictions. Here, we focus on efficient and scalable numerical solvers for coupled single-phase flow and mechanical processes in geological formations based on a three-field mixed finite-element (FE) discretization of the governing equations.

From a discretization point of view, a mixed FE scheme imposes an inf-sup (or LBB) compatibility constraint on the selection of the discrete spaces for interpolating the primary variables fields [16]. Violation of this constraint may result in numerical solutions that exhibit different forms of instabilities. For example, a classical two-field displacement-pressure formulation based on the continuous Galerkin FE method—the most popular technique used in consolidation modeling [17]—typically generates nonphysical pressure oscillations, when incompressible or impermeable conditions are approached, if an equal-order interpolation is used for both discrete variables. Stable discretizations may be achieved by either selecting discrete spaces that are intrinsically LBB-stable, or devising suitable stabilization techniques. The classical Taylor-Hood elements [18] belong to the first class. Examples of stabilized formulations, which allow for preserving the advantages of using equal-order interpolation, are proposed in [19, 20].

*Corresponding author
Email addresses: `ncastell@stanford.edu` (Nicola Castelletto), `jawhite@llnl.gov` (Joshua A. White), `massimiliano.ferronato@unipd.it` (Massimiliano Ferronato)