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Stable evaluation of Green's functions in cylindrically stratified regions with uniaxial anisotropic layers

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ABSTRACT

We present a robust algorithm for the computation of electromagnetic fields radiated by point sources (Hertzian dipoles) in cylindrically stratified media where each layer may exhibit material properties (permittivity, permeability, and conductivity) with uniaxial anisotropy. Analytical expressions are obtained based on the spectral representation of the tensor Green's function based on cylindrical Bessel and Hankel eigenfunctions, and extended for layered uniaxial media. Due to the poor scaling of these eigenfunctions for extreme arguments and/or orders, direct numerical evaluation of such expressions can produce numerical instability, i.e., underflow, overflow, and/or round-off errors under finite precision arithmetic. To circumvent these problems, we develop a numerically stable formulation through suitable rescaling of various expressions involved in the computational chain, to yield a robust algorithm for all parameter ranges. Numerical results are presented to illustrate the robustness of the formulation including cases of practical interest.

1. Introduction

Analysis of electromagnetic fields in cylindrically stratified media is of great importance in many applications, such as borehole geophysics [1–3]. This is a classical problem with separable geometry where the components of the tensor Green's function can be expressed in generic form as [4, Ch. 3],[5]

$$\sum_{n=-\infty}^{\infty} e^{\mathrm{i}n(\phi-\phi')} \int_{-\infty}^{\infty} dk_z e^{\mathrm{i}k_z(z-z')} \mathbf{\Phi}_n(\rho,\rho'), \tag{1}$$

where the integrand factor $\mathbf{\Phi}_n(\rho,\rho')$ contains various products of cylindrical Bessel and Hankel functions. When applicable, such solutions are often preferred to brute-force numerical methods such as finite elements and finite difference [6–15] since the former can provide very accurate results with computational costs that are orders of magnitude smaller than the latter. This is especially important for inverse algorithms relying on repeated forward solutions and which seek to determine sought-after physical parameter values (say, layer resistivities) from the knowledge of the field values (measured) at certain subterranean locations.

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However, numerical computations directly based on the canonical expressions of this problem can lead to underflow and overflow issues in finite precision arithmetic. This is caused by the poor scaling of cylindrical Bessel and Hankel functions for extreme arguments and/or orders, which occur for low frequencies of operation and/or extreme values for layer resistivities. In addition, convergence problems in the numerical evaluation of the spectral integral on the longitudinal wavenumber k_z may occur depending on the separation distance between the source (ρ', ϕ', z') and observation point (ρ, ϕ, z) as well as on the operation frequency. To circumvent these problems, a stable formulation based on a suitable analytical conditioning of the various factors in the computational chain and a proper choice of deformed integration paths in the complex k_z plane was recently put forth in [5]. This formulation was shown to be robust to variations on physical parameters that span several orders of magnitude. A related formulation to compute static fields (electric potentials) due to current electrodes in isotropic layers was described in [16].

In this work, we extend the formulation presented in [5] to account for scenarios where the layers comprising the cylindrical stratified media may exhibit anisotropic properties. In borehole geophysics, anisotropy is quite common [17–37] and may result from geological factors affecting the various Earth layers such as salt water penetrating porous fractured formations and thereby increasing the conductivity in the direction parallel to the fracture and/or the presence of clay and sand laminates with directionally dependent resistivities. Here, for generality, we assume each layer to be doubly uniaxial, i.e., both the complex permittivity tensor $\overline{\overline{\epsilon}}$ (which includes the conductivity tensor) and the permeability tensor $\overline{\overline{\mu}}$ are independently uniaxial, which facilitates the analysis of equivalent problems using electromagnetic duality [4, Ch. 1].

2. Fields in cylindrically-layered uniaxial media

Most of the basic notation and terminology is adopted from [4, Ch. 3]. The section can be regarded as a generalization of the formulation presented for isotropic layers in [5] to uniaxial anisotropic layers.

2.1. General solution in homogeneous, uniaxial media

Maxwell's curl equations in uniaxial, homogeneous, and source-free media (with time-harmonic dependence $e^{-i\omega t}$ assumed) read as

$$\nabla \times \mathbf{E} = i\omega\overline{\overline{\mu}}\mathbf{H}, \tag{2}$$

$$\nabla \times \mathbf{H} = -i\omega\overline{\overline{\epsilon}}\mathbf{E}, \tag{3}$$

where $\overline{\overline{\mu}}$ and $\overline{\overline{\epsilon}}$ are the permeability tensor and complex permittivity tensor, respectively. In the unixial case, $\overline{\overline{\mu}}$ is written as

$$\overline{\overline{\mu}} = \begin{bmatrix} \mu_h & 0 & 0 \\ 0 & \mu_h & 0 \\ 0 & 0 & \mu_v \end{bmatrix}, \tag{4}$$

where μ_h and μ_v are the horizontal and vertical permeabilities, resp. The complex permittivity tensor $\overline{\overline{\epsilon}}$ includes the electric conductivity and it is written as

$$\overline{\overline{\epsilon}} = \begin{bmatrix} \epsilon_h & 0 & 0 \\ 0 & \epsilon_h & 0 \\ 0 & 0 & \epsilon_v \end{bmatrix} = \begin{bmatrix} \epsilon_{p,h} + i\sigma_h/\omega & 0 & 0 \\ 0 & \epsilon_{p,h} + i\sigma_h/\omega & 0 \\ 0 & 0 & \epsilon_{p,v} + i\sigma_v/\omega \end{bmatrix}, \tag{5}$$

where $\epsilon_{p,h}$ and $\epsilon_{p,v}$ are horizontal and vertical permittivities, and σ_h and σ_v are horizontal and vertical conductivities, resp. In such source-free media, the divergence equations can be written as

$$\nabla \cdot \left(\overline{\overline{\epsilon}} \cdot \mathbf{E}\right) = 0, \tag{6}$$

$$\nabla \cdot \left(\overline{\overline{\mu}} \cdot \mathbf{H}\right) = 0. \tag{7}$$

Note that in general $\nabla \cdot \mathbf{E}$ and $\nabla \cdot \mathbf{H}$ in uniaxial and source-free media are nonzero. Indeed, the left hand side of (6) in cylindrical coordinates is written as

$$\nabla \cdot \overline{\overline{\epsilon}}\mathbf{E} = \epsilon_h \left\{ \frac{1}{\rho}\frac{\partial\left(\rho E_\rho\right)}{\partial \rho} + \frac{1}{\rho}\frac{\partial E_\phi}{\partial \phi} + \frac{\partial E_z}{\partial z} - \left(1 - \frac{\epsilon_v}{\epsilon_h}\right)\frac{\partial E_z}{\partial z} \right\} = \epsilon_h \left\{ \nabla \cdot \mathbf{E} - \left(1 - \frac{\epsilon_v}{\epsilon_h}\right)\frac{\partial E_z}{\partial z} \right\}. \tag{8}$$

From (6) and (8), we can obtain

$$\nabla \cdot \mathbf{E} = \left(1 - \frac{\epsilon_v}{\epsilon_h}\right)\frac{\partial E_z}{\partial z}. \tag{9}$$