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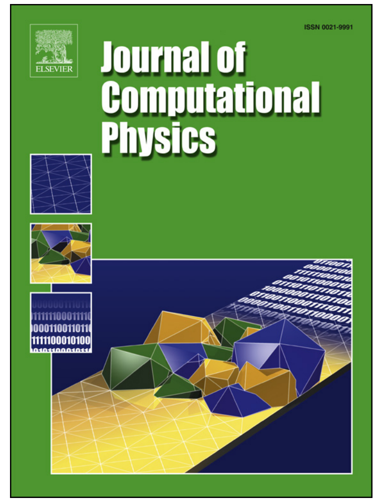

A fully conservative mimetic discretization of the Navier–Stokes equations in cylindrical coordinates with associated singularity treatment

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A fully conservative mimetic discretization of the Navier-Stokes equations in cylindrical coordinates with associated singularity treatment

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Abstract

We present a finite difference discretization of the incompressible Navier-Stokes equations in cylindrical coordinates. This currently is, to the authors' knowledge, the only scheme available that is demonstrably capable of conserving mass, momentum and kinetic energy (in the absence of viscosity) on both uniform and non-uniform grids. Simultaneously, we treat the inherent discretization issues that arise due to the presence of the coordinate singularity at the polar axis. We demonstrate the validity of the conservation claims by performing a number of numerical experiments with the proposed scheme, and we show that it is second order accurate in space using the Method of Manufactured Solutions.

Keywords: incompressible flow, cylindrical coordinates, mimetic finite difference method, kinetic energy conservation

1. Introduction

Although it is generally known that the use of cylindrical coordinates in finite difference methods brings along a number of difficulties, it still appears to be the preferred method of choice for turbulent flow simulations in pipe sections. This is likely due to the relative ease with which higher order approximations can be implemented, and the growing availability of fast flow solvers that benefit from the orthogonality of the structured cylindrical grid. However, an inherent problem in the use of cylindrical coordinates (r, θ, z) is the calculation of variables that lie on or near the polar axis $r = 0$. Looking at the cylindrical Navier-Stokes equations :

$$\frac{1}{r}\frac{\partial(ru_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0, \quad (1)$$

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