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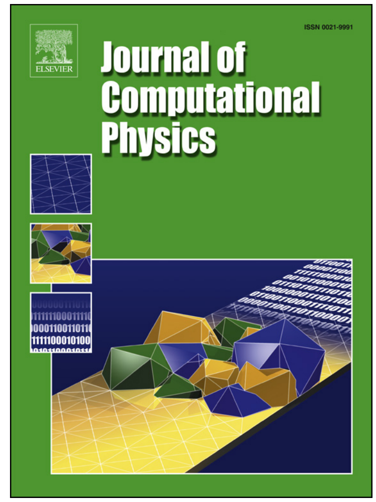

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Gauss-Jacobi quadratures for weakly, strongly, hyper- and nearly-singular integrals in boundary integral equation methods for domains with sharp edges and corners

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Abstract

We present Gauss-Jacobi quadrature rules in terms of hypergeometric functions for the discretization of weakly singular, strongly singular, hypersingular, and nearly singular integrals that arise in integral equation formulations of potential problems for domains with sharp edges and corners. The rules are tailored to weight functions with algebraic endpoint singularities of a fairly general form, thus allowing one to easily incorporate a wide class of domains into the analysis. Numerical examples illustrate the accuracy and stability of the proposed algorithms; it is shown that the same level of high accuracy can be achieved for any choice of the external variable. The usefulness of the method is exemplified by application to the solution of a singular integral equation that arises in time-harmonic electromagnetic scattering by either closed or open perfectly conducting cylindrical objects with edges and corners, such as polygon cylinders and bent strips. Some practical aspects concerning the role of nearby singularities in achieving a highly accurate solution of singular integral equations are, also, discussed.

Keywords: weakly singular integrals, strongly singular integrals, hypersingular integrals, nearly singular integrals, Gauss-Jacobi quadrature rules, Nyström method

1. Introduction

Integrals of the type

$$I^{(m)}(t) = \int_{-1}^{1} w(\tau) f(\tau) \frac{\partial^m \ln|\tau - t|}{\partial t^m} d\tau, \quad -1 \leq t \leq 1, \quad m = 0, 1, 2 \tag{1}$$

having integrable endpoint singularities of the form

$$w(\tau) = (1-\tau)^{\alpha}(1+\tau)^{\beta} \quad (\alpha > -1,\ \beta > -1) \tag{2}$$

arise in a natural way in integral equation formulations of potential problems for domains with sharp edges and corners. In physical problems, the singularities at the endpoints $\tau = \pm 1$ are imposed by physical constraints usually referred to as the edge conditions, which are related to the geometrical singularities of the domain. Obviously, the integral $I^{(m)}(t)$ does not exist for $m = 1, 2$ and must therefore be understood in a non-classical sense, namely, as a Cauchy-type