Journal of Informetrics 11 (2017) 685–688

Contents lists available at ScienceDirect

Journal of Informetrics

journal homepage: www.elsevier.com/locate/joi

Correspondence

Counting citations: Generalizing the Perry-Reny index

1. Introduction

In a recent paper in a prominent journal in the field of economics, Perry and Reny (2016) propose an important index for counting citations. In contrast to previously used and proposed indices that are 'ad hoc measures based almost entirely on intuition and rules of thumb with insufficient justification' [p. 2723; all unnamed page numbers refer to Perry and Reny (2016)] and 'can produce intuitively implausible rankings' (p. 2722), the Perry-Reny index is derived from (at least mostly) compelling axioms. It thus provides a very important advance in the area of counting citations. This note proposes a slight change in the Perry-Reny index, making it more general. In particular, the generalized index is capable of allowing for different degrees of weighting depth and/or width. The generalization is partly based on the non-acceptance of their less compelling Axiom 5 and the accounting of a tiny ambiguity (regarding linear homogeneity) in their argument, and partly based on the desirability of allowing for different degrees of weighting depth and/or width.

Though the generalized index is already discussed in by Perry and Reny (2016), their proposed index is a special case of the general index and is not capable of allowing for different degrees of weighting depth and/or width. The proposed generalization is thus of some importance. Section 2 of this note explains the ambiguity and the generalization. Section 3 reports a simple empirical survey showing that the preferred weighting of depth and/or width may not be the same as the one implied by the Perry-Reny index. However, the proposed generalized Perry-Reny index covers all different degrees of this weighting.

2. The Perry-Reny index and its generalization

Based on four compelling axioms and a fifth debatable one, Perry and Reny (2016) show that any citation index satisfying the axioms must be equivalent to their proposed index (which they call the Euclidean Index) that assigns any citation list, $(x_1, \ldots, x_n)$, the number

$$\left[\sum_i (x_i^2)\right]^{1/2}. \tag{1}$$

where x_i is the citation count of paper i. The four compelling axioms include: Monotonicity, Independence, Depth Relevance, and Scale Invariance. These four compelling axioms only imply that the index must be equivalent to the σ-index (also called the generalized Perry-Reny index here):

$$\left[\sum_i (x_i^\sigma)\right]^{1/\sigma}. \tag{2}$$

Obviously, for the special case of $\sigma = 2$, we have the Perry-Reny index in (1). Thus, the σ-index in (2) above is the generalized Perry-Reny index proposed in this note which is more useful, as argued below.

Perry and Reny appear to justify the confinement to the specific index (1) on both their Axiom 5 Directional Consistency (discussed below) and the homogeneity of degree one of the index when $\sigma = 2$. This linear homogeneity implies that doubling the citation numbers of all papers doubles the overall citation index. This is convenient; 'For example, suppose that the Euclidean index of x is twice that of y, . . . How might one describe how the two lists compare to one another in terms that would be helpful to an administrator or someone outside the field in question? The answer is that one could say that "list x is as good as the list that has as many papers as in y, but receives twice as many citations on each of them"' (p. 2728). This is indeed convenient. However, Perry and Reny mention that, of all indices satisfying the compelling axioms, 'only the

http://dx.doi.org/10.1016/j.joi.2017.05.008

Euclidean index [i.e. with $\sigma = 2$] . . . and its positive multiples are homogeneous of degree 1′ (p. 2728). In fact, the σ-index is homogeneous of degree one for all values of σ. To show this, we multiply all x_i in (2) by a common constant λ to get:

$$\left[\sum_i \left(\lambda x_i\right)^{\sigma}\right]^{1/\sigma} = \left[\sum_i \left(\lambda^{\sigma} x_i^{\sigma}\right)\right]^{1/\sigma} = \left[\lambda^{\sigma}\sum_i \left(x_i^{\sigma}\right)\right]^{1/\sigma} = \lambda\left[\sum_i \left(x_i^{\sigma}\right)\right]^{1/\sigma}$$

which establishes the linear homogeneity of the generalized Perry-Reny index (2) for all values of σ. Thus, we do not have to be confined to $\sigma = 2$ to have the desirable linear homogeneity.[1]

Another reason Perry and Reny opt for the Euclidean index instead of the more general σ-index is their Axiom 5. They motivate and state the axiom thus:

‘Consider two individuals with equally ranked citation lists who, over the next year, each receive the same number of additional citations on their most cited paper, and the same number but fewer additional citations on their second most cited paper, etc. Suppose that at the end of the year their lists remain equally ranked. Directional consistency requires that if their papers continue to accrue citations at those same yearly rates, then their lists will continue to remain equally ranked year after year.’

While the meaning of this axiom is clear, its compellingness is unclear. It is apparent that Perry and Reny also regard their Axioms 1–4 as more compelling than this Axiom 5, as they call the former ‘basic properties’, and the latter only ‘property’ (p. 2725 and p. 2727). Without Axiom 5, the value of σ need not be confined to 2.[2]

The value of σ may be seen to be the extent depth is valued relative to width. Take the simple case of just two papers for both scholars (similar in all other aspects for ease of comparison). Consider the citation profile of (100, 0) (i.e. one of the two papers has been cited 100 times, and the other not cited at all; ignoring the order or time profile of publication) versus that of (50, 50) (i.e. both papers cited 50 times each). We also ignore differences in quality or assume that the citation numbers have been quality-adjusted (thus also allowing for fractional counts, as allowed in Perry and Reny). If depth is valued as much as width, the two profiles are regarded as equivalent in terms of desirability and should have the same overall citation index. This is achieved in (2) by having $\sigma = 1$, reducing the generalized Perry-Reny index into the index of total citation count. If depth is regarded as more important than width, then (100, 0) is preferable to (50, 50). This may be allowed in (2) by having $\sigma > 1$. Conversely, if width is regarded as more important than depth, (50, 50) is preferable to (100, 0). This is allowed in (2) by having $\sigma < 1$. Perry and Reny may be right in believing that depth is more important than width by requiring their Axiom 3 of Depth Relevance. This justifies only that σ should be larger than one; it needs not be as large as 2, as required by their Euclidean index, or the Perry-Reny index before the generalization proposed here.

The value of $\sigma = 2$ suggests that depth is very much more important than width. For example, it suggests that, for a scholar with two equally cited papers to have an equivalent citation index as (100, 0), (50, 50) is not enough. Rather, it takes (70.71, 70.71) to have an equivalent index as (100, 0). As the relative importance of depth versus width is largely a matter of opinion, this high degree on the importance of depth may be regarded as reasonable by some, but perhaps not by the overwhelming majority. The result of the illustrative and non-definitive survey reported in the next section suggests that a clear majority prefers a value of σ significantly less than 2. Thus, allowing the value of σ to be variable should be more general, as it allows different degrees of depth-width weighting to be entertained. In fact, by having $\sigma < 1$, the generalized Perry-Reny index (2) allows even for cases where width is regarded as more important than depth, though this violates their Axiom 3, which requires $\sigma > 1$.

Recent work by Andersen (2017) examines the Euclidean index in detail and compares it to some other indices both conceptually and empirically. Though not focusing on the depth-width issue, Andersen also appears to have some hesitation on whether the degree of depth relevance should be as high as having $\sigma = 2$: ‘it is also well-known that there is a Matthew effect leading to an overemphasis of papers that are already highly cited’ (Andersen, 2017, p. 457).

Actually, Perry and Reny also undertake some empirical studies, based on a dataset constructed by Ellison (2013). They show that both the Euclidean index and the general σ-index by far outperform the h-index in matching labor market data. It is interesting to note that the σ value that maximizes the matching is 1.85 (p. 2734). Though this ‘is remarkably close to the value of $\sigma = 2$ that defines the Euclidean index’ (p. 2734), it is also significantly less than 2. Moreover, since the data are confined to the top 50 research universities, the figure of 1.85 may already reflect some bias, as leading researchers with some heavily cited papers may tend to prefer a higher σ value. If the views of academics in other universities and teaching colleges are included, a lower value of σ may be preferred, as Reny noted to me in a private correspondence. Since the best value of σ is largely a matter of personal preference, I have undertaken a simple empirical study to find out the average and different preferred values from a selected sample of academics, as reported in the next section.

[1] In reading their paper, I got the impression that Perry and Reny justify the confinement to $\sigma = 2$ partly on its linear homogeneity. However, in a private correspondence, Reny told me that they ‘do not *formally* justify the choice of $\sigma = 2$ by way of the homogeneity property’ (italics added), but rather by axiom 5.

[2] The formal statement of Axiom is: (v) *Directional Consistency*.—$\forall\, x, y, d \in L$, if $\iota(x) = \iota(y)$ and $\iota(x+d) = \iota(y+d)$, then $\iota(x+\lambda d) = \iota(y+\lambda d)$ for all $\lambda > 1$.’ (p. 2727). As x and y involve different profiles of possibly different degrees of evenness or disparity, e.g. one may be (100, 0) and the other may be (70, 50), the successive addition of the same d or multiple of d may not satisfy Axiom 5 without being inconsistent in the ranking. In fact, this may depend on the degree the ranker gives to the importance of depth vs. width.