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Nonlinear systems control using self-constructing wavelet networks

Cheng-Jian Lin [a,b,*]

[a] Department of Electrical Engineering, National University of Kaohsiung, Kaohsiung City 811, Taiwan, ROC

[b] Department of Computer Science and Information Engineering, National Chin-Yi University of Technology, Taichung County 411, Taiwan, ROC

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ABSTRACT

This paper describes a self-constructing wavelet network (SCWN) controller for nonlinear systems control. The proposed SCWN controller has a four-layer structure. We adopt the orthogonal wavelet functions as its node functions. An online learning algorithm, structure learning and parameter learning, allows the dynamic determining of the number of wavelet bases, and adjusting the shape of the wavelet bases and the connection weights. The SCWN controller is a highly autonomous system. Initially, there are no hidden nodes. They are created and begin to grow as learning proceeds. Computer simulations have been conducted to illustrate the performance and applicability of the proposed learning scheme.

1. Introduction

Nonlinear system control is becoming an important tool, which can be used to improve control performance and achieve robust fault-tolerant behavior. Among the different nonlinear control techniques, methods based on artificial neural network (ANN) have been grown into a popular research topic in recent years [1–3]. The reason is that the classical control theory usually requires a mathematical model for designing the controller. The inaccuracy of mathematical modeling of the plants usually degrades the performance of the controller, especially for nonlinear and complex control problems [4]. ANN modeling has been admitted as a powerful tool, which can facilitate the effective development of models by combining information from different sources, such as data, records. However, the ANN lacks a systematic way to determine the appropriate model structure, has no localizability, and converges slowly. A suitable approach to overcoming the disadvantages of global approximation networks is the substitution of the global activation function with localized basis functions. In this type of local network, only a small subset of the network parameters is engaged at each point in the input space. The network transparency may be improved by adopting the wavelet decomposition technique from the field of adaptive signal processing. Due to the local properties of wavelets, arbitrary functions can be approximated by the truncated discrete wavelet transform.

Recently, many researches proposed wavelet neural networks for identification and control [5–14]. Ikonomopoulos and Endou [9] proposed the analytical ability of the discrete wavelet decomposition with the computational power of radial basis function networks. Members of a wavelet family were chosen through a statistical selection criterion that constructs the structure of the network. Ho et al. [10] used the orthogonal least squares (OLS) algorithm to purify the wavelets from their candidates, which avoided using more wavelets than required and often resulted in an overfitting of the data and a poor situation in ref. [6]. Lin et al. [11] proposed a wavelet neural network to control the moving table of a linear ultrasonic motor (LUSM) drive system. They chose an initialization for the mother wavelet based on the input domains defined by the examples of the training sequence. Huang and Huang [12] proposed an evolutionary algorithm for optimally adjusted wavelet networks. However, the selections of wavelet bases were based on practical experience or trial-and-error tests.

To steady control the nonlinear systems, a self-constructing wavelet network (SCWN) controller is proposed in this paper. It is a four-layered network structure, which is comprised of an input layer, wavelet layer, product layer, and output layer. We adopt the orthogonal wavelet functions as its node functions. Based on the self-learning ability, the on-line structure/parameter learning algorithm is performed concurrently in the SCWN controller. In the structure learning scheme, the degree measure method is used to find the proper wavelet bases and to minimize the number of wavelet bases generated from input space. In parameter learning scheme, the supervised gradient descent method is applied to adjust the shape of wavelet functions and the connection weights

* Correspondence address: Department of Electrical Engineering, National University of Kaohsiung, Kaohsiung City 811, Taiwan, ROC.
E-mail address: cjlin@nuk.edu.tw.

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in SCWN controller. To a great extent, the learning quality of a network is related to the parameters of feedback error. The SCWN controller based on feedback error learning strategy has favorable control performance. Finally, the proposed SCWN controller is applied to two nonlinear control problems: control for backing up the truck, and control of water bath temperature system. The proposed SCWN model has the following advantages: (1) this study adopts the wavelet network to control nonlinear systems. The local properties of wavelets in the SCWN model enable arbitrary functions to be approximated more effectively. (2) We use an online learning algorithm to automatically construct the SCWN model. No nodes or wavelet bases exist initially. They are created automatically as learning proceeds, as online incoming training data are received and as structure and parameter learning are performed. The structure learning adopts partition-based clustering techniques to perform cluster analysis in a data set. The parameter learning, based on the gradient descent method, can adjust the wavelet functions and the corresponding weights of the SCWN. (3) As demonstrated in Section 4, the SCWN model is characterized by small network size and fast learning speed.

2. Structure of the SCWN controller

The structure of the SCWN controller is shown in Fig. 1. The proposed SCWN controller is designed as a four-layer structure, which is comprised of an input layer, wavelet layer, product layer, and output layer.

The input data in the input layer of the network is $x = [x_1, x_2, \ldots, x_i, \ldots, x_n]^T$, where T is the transpose and n is the number of dimensions. Noted that in ordinary wavelet neural network model applications, it is often useful to normalize the input vectors x into the interval [0,1]. Then, the activation functions of the wavelet nodes in the wavelet layer are derived from the mother wavelet $\phi(x)$, with a dilation of d and a translation of t [6]:

$$\phi_{d.t}(x) = 2^{d/2}\phi(2^d x - t) \tag{1}$$

The mother wavelet is selected so that it constitutes an orthonormal basis in $L^2(\Re^n)$. The derivation of a differentiable *Mexican-hat* function is adopted as a mother wavelet herein,

$$\phi(x) = (1 - ||x||^2)\,\mathrm{e}^{-||x||^2/2}, \tag{2}$$

where $||x||^2 = x^T x$. Therefore, the activation function of the jth wavelet node connected with the ith input data is represented as:

$$\phi_{d_j t_j}(x_i) = 2^{d_{ij}/2}(1 - ||2^{d_{ij}} x_i - t_{ij}||^2)\,\mathrm{e}^{-||2^{d_{ij}} x_i - t_{ij}||^2/2}, \quad i = 1, \ldots, n; \quad j = 1, \ldots, m, \tag{3}$$

where n is the number of input-dimensions and m is the number of the wavelets. The wavelet functions of (3) with various dilations and translations are presented in Fig. 2. Then, each wavelet in the product layer is labeled Π, i.e., the product of the jth multi-dimensional wavelet with n input dimensions $x = [x_1, x_2, \ldots, x_i, \ldots, x_n]^T$ can be defined as

$$\psi_j(x) = \prod_{i=1}^{n} \phi_{d_j t_j}(x_i). \tag{4}$$

According to the theory of multi-resolution analysis (MRA) [10,13], any $f \in L^2(\Re)$ can be regarded as a linear combination of wavelets at different resolution levels. For this reason, the function f is expressed as

$$Y(x) = f(x) \approx \sum_{j=1}^{m} w_j \psi_j(x) \tag{5}$$

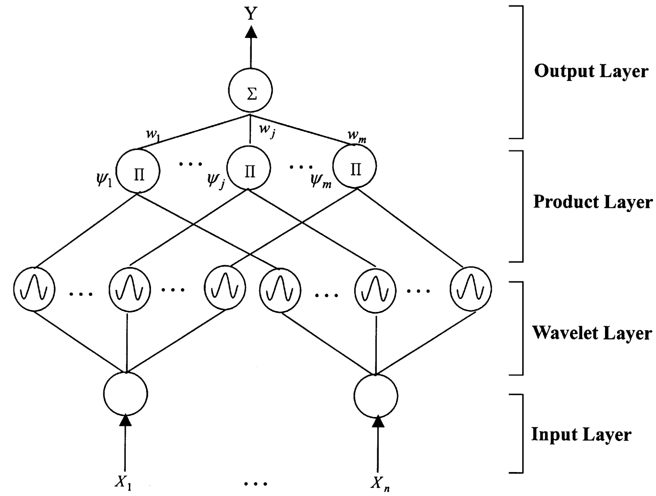

Fig. 1. The architecture of the SCWN controller.

If $\psi_j = [\psi_1, \psi_2, \ldots, \psi_m]$ is used as a nonlinear transformation function of hidden nodes and weight vectors and $w_j = w_1, w_2, \ldots, w_m$ defines the connection weights, then Eq. (5) can be considered the functional expression of the SCWN modeling function Y.

3. A self-constructing learning algorithm

In this section, the *degree measure* method and the well-known *back propagation* (BP) algorithm are used concurrently for constructing and adjusting the SCWN controller. The degree measure method is used to decide the number of wavelet bases in the wavelet layer and the product layer. On the other hand, the BP algorithm is used to adjust the parameters of the wavelet bases and connection weights. The details of the algorithm are presented below. Finally in this section, the stability analysis of the SCWN model based on the Lyapunov approach is performed the convergence property.

3.1. The structure learning scheme

Initially, there are no wavelet bases in the SCWN controller. The first task is to decide when a new wavelet base is generated. We adopt partition-based clustering techniques to perform cluster analysis in a data set. For each incoming pattern x_i, the firing strength of a wavelet base can be regarded as the degree of the incoming pattern belonging to the corresponding wavelet base. An input datum x_i with a higher firing strength means that its spatial location is nearer to the center of the wavelet base t_j than those with smaller firing strength. Based on this concept, the firing strength obtained from Eq. (4) in the product layer can be used as the degree measure

$$F_j = |\psi_j|, \quad j = 1, \ldots, q, \tag{6}$$

where q is the number of existing wavelet bases and $|\psi_j|$ is the absolute value of ψ_j. According to the degree measure, the criterion of a new wavelet base generated for new incoming data is described as follows:

Find the maximum degree $F_{\max}$

$$F_{\max} = \max_{1 \le j \le q} F_j \tag{7}$$

If $F_{\max} \le \bar{F}$, then a new wavelet base is generated, where $\bar{F}$ is a pre-specified threshold that should decay during the learning process, limiting the size of the SCWN model.