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Variable speed limit design based on mode dependent Cell Transmission Model☆

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ABSTRACT

In this paper a mode dependent variable speed limit (VSL) control strategy is developed for motorway networks. The suggested rolling horizon and coordinated algorithm uses switching mode Cell Transmission Model (CTM) and purports to maximize network throughput.

In this line, first, a VSL signal scheduled piecewise affine switching mode CTM is derived based on the polyhedral description of Godunov fluxes. Second, a two-stage, coordinated, rolling horizon VSL sequence generation procedure is proposed. The set of possible VSL signs is selected by applying input constraints in order to eliminate spatial and temporal VSL oscillations. Then, the set of modes is further reduced according to the *stable* and adjacent reachable modes of the switching mode CTM. Over the remaining set of input signals, network capacity is maximized with the help of solving a mixed integer optimization problem under the form of reference density tracking objective. The method is implemented in simulation environment to demonstrate its computational efficiency and viability to attenuate shockwaves.

1. Introduction

The continuously growing road transport demand may traditionally be addressed by expanding the available road infrastructure, i.e. road construction. This physical expansion of the network is not only costly or sometimes very challenging (e.g. in urban areas), but also may adapt to changing traffic conditions in a rather conservative way. Hence, Intelligent Transport Systems (ITS) have been applied as an alternative to track the dynamic demand patterns. In motorway networks, such efficiency (and safety) indicators try to compensate traffic congestion in order to decrease travel delay, emission, energy consumption and crash risk, etc.

In order to capture the macroscopic and dynamic traffic patterns on motorways, different traffic flow models have been proposed. A first order model, governed by the vehicle conservation law and including the fundamental relationship between flow and density was suggested by Lighthill and Whitham (1955) and Richards (1956). Based on the discretization scheme of Godunov (1959), a numerical solution is given to the LWR model resulting in the well-known Cell Transmission Model (CTM) by Daganzo (1994). CTM is convenient in terms of describing capacity changes since it is capable of interpreting kinematic (congestion and rarefaction) waves.

Assuming a triangular form for the fundamental diagram, the model can be reformulated as switching mode dynamics belonging to the class of piecewise affine model representations, see Sontag (1981). This form is very attractive for model-based traffic management (traffic state estimation and control), for two main reasons: (i) scalability and computational burden (ii) it is control theory supported, Borrelli et al. (2011).

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With this switching concept in mind network state estimation problem has been considered in Kurzhanskiy (2009), Canudas De Wit et al. (2012), Morbidi et al. (2014), Vivas et al. (2015), and Sun and Work (2014) via modes associated to each cell by distinguished Godunov flux pairs. An effective solution to mode-related traffic state estimation is proposed by means of wavefront localization: in case of N segments, $N + 1$ operation modes are defined, see Munoz et al. (2003) and Morarescu and Canudas De Wit (2011). A joint estimation-ramp control problem is proposed in Sun et al. (2003). The approach defines two main controller modes: all-congested and all-free-flow cell conditions. In Thai and Bayen, 2013, 2015 an explicit piecewise affine decomposition of the Godunov scheme is formulated for network state estimation of uncontrolled systems, leading to a polyhedral description of operation modes. This approach provides a complete description of the system modes, for the entire state domain of each cell.

Motorway control measures - i.e. ramp metering and variable speed limit (VSL) control - need special modelling efforts when applying to the CTM. Mode dependent CTM has been formalized for (mostly local) ramp metering but only a few studies deal with speed limit control solutions. In the works Jacquet et al. (2006) and Lemarchand et al. (2011), robust ramp meter design approaches are suggested. Similarly to the previous works, the number of switching modes are determined with the help of jamwave front localization. Furthermore, the appropriate restriction of boundary conditions can lead to efficient switching mode based ramp solution, see Gomes et al. (2008). Focusing on the ramp metering problem, control-oriented switching mode representations are given in Pisarski and de Wit (2012) and Ferrara et al. (2015). In both cases, the operation modes are selected including the effect of ramp metering over the traffic flow conditions resulting in a rule-based control solution.

Variable speed limit control has only been considered in a limited number of papers with CTM. With mode dependent CTM, Muralidharan and Horowitz (2015) divides the coordinated ramp metering-VSL problem to several subproblems, according to the flow conditions at the cell interfaces. Each problem is then relaxed and solved as an effective linear programming problem. An online model predictive control (MPC) approach for VSL inclusion is presented in Hadiuzzaman and Tony (2013). In that work no switching mode description is used, instead, CTM is treated as a nonlinear model due to the minimum conditions of Godunov flows. A thorough discussion of speed limit modeling with the triangular fundamental diagram is given by Han et al. (2017). The proposed VSL model is applied for CTM, and the minimum conditions of the CTM are used to define a relaxed linear problem in Han et al. (2017).

Using the shockwave theory, the method SPECIALIST by Hegyi et al. (2008) proposes an easily implementable time-update free, feedforward control solution for coordinated VSL. Our proposed controller scheme uses similar concept, however, a recursive VSL is considered via the dynamics of CTM.

The aim of the present paper is to develop a systematic methodology for motorway traffic flow control through a switching mode CTM-based VSL design algorithm. In this line, our contribution is twofold. First, as an extension of the works by Thai and Bayen (2013, 2015), the inclusion of VSL into the framework of polyhedral CTM description is shown. In the derived form the system matrices as well as the polyhedral representation of the system are triggered by the control signal, i.e. an (input) parameter-dependent PWA model is obtained. Second, a coordinated VSL control algorithm is proposed. After reducing the set of input signals by means of spatial and temporal constraints (similarly as in Frejo et al. (2014)), an analysis of reachable modes is carried out and those signals are kept that provide reachability of stable modes for the highest number of segments. Over the remaining set of control signals an optimization is carried out to maximize network throughput on a finite prediction horizon.

With the current technique developed we intend to show that an appropriate handling of the VSL included CTM enables a very effective control for shockwave suppression. During control design, the input dependence of the polyhedral representation of the system is exploited. Numerical results affirm that the analysis of reachable modes during control set reduction has a key role in obtaining a controller with a fast response and low computational demand.

The layout of the paper is as follows. In Section 2, the CTM is reviewed alongside its polyhedral representation, summarizing the modelling contributions of Thai and Bayen (2013, 2015). Then, following the modelling assumptions, the polyhedral representation of the CTM is given for VSL controlled networks in Section 3. Control design is detailed in Section 4. Simulation results are shown in Section 5.

2. Preliminaries

2.1. The continuous LWR model

Lighthill and Whitham (1955) and Richards (1956) introduced a macroscopic dynamic model of traffic based on the conservation of vehicles:

$$\frac{\partial\rho(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = r(x,t) - s(x,t), \tag{1}$$

where $\rho(x,t)$ and $q(x,t)$ denote the density and the flow of vehicles at location x and time t, respectively. Additional sources and sinks of traffic are represented by $r(x,t)$ and $s(x,t)$. In practice, these flows come from on- and off-ramps. The model adopts the hypothesis of Greenshields (1935), assuming a static flow-density relationship, known as the flow function:

$$q(x,t) = Q(\rho(x,t)). \tag{2}$$

Several flow functions have been suggested. The commonly used triangular flow function (see Fig. 1a) is given in the form

$$Q(\rho) = \begin{cases} v_f\rho & \text{if } \rho \leqslant \rho_{cr} \\ -w(\rho-\rho_{jam}) & \text{if } \rho > \rho_{cr} \end{cases}, \tag{3}$$

where ρ_{cr} and ρ_{jam} denote the critical and maximal traffic density respectively, and w denotes the backward propagating wave speed.