Transportation Research Part C 82 (2017) 102–123

Contents lists available at ScienceDirect

Transportation Research Part C

journal homepage: www.elsevier.com/locate/trc

Nonlinear programming methods based on closed-form expressions for optimal train control

Hongbo Ye, Ronghui Liu *

Institute for Transport Studies, University of Leeds, Leeds LS2 9JT, United Kingdom

ARTICLE INFO

Article history:
Received 22 November 2016
Received in revised form 5 June 2017
Accepted 15 June 2017

Keywords:
Optimal train control
Energy consumption
Nonlinear programming
Closed-form expression
Subway line scheduling
Simultaneous multi-train control

ABSTRACT

This paper proposes a novel approach to solve the complex optimal train control problems that so far cannot be perfectly tackled by the existing methods, including the optimal control of a fleet of interacting trains, and the optimal train control involving scheduling. By dividing the track into subsections with constant speed limit and constant gradient, and assuming the train's running resistance to be a quadratic function of speed, two different methods are proposed to solve the problems of interest. The first method assumes an operation sequence of maximum traction – speedholding – coasting – maximum braking on each subsection of the track. To maintain the mathematical tractability, the maximum tractive and maximum braking functions are restricted to be decreasing and piecewise-quadratic, based on which the terminal speed, travel distance and energy consumption of each operation can be calculated in a closed-form, given the initial speed and time duration of that operation. With these closed-form expressions, the optimal train control problem is formulated and solved as a nonlinear programming problem. To allow more flexible forms of maximum tractive and maximum braking forces, the second method applies a constant force on each subsection. Performance of these two methods is compared through a case study of the classic single-train control on a single journey. The proposed methods are further utilised to formulate more complex optimal train control problems, including scheduling a subway line while taking train control into account, and simultaneously optimising the control of a leader-follower train pair under fixed- and moving-block signalling systems.

1. Introduction

The traditional optimal train control problem is to find the energy-efficient strategy to drive the train through a specific railway segment within a predetermined time, while maintaining specific speeds at both ends of the segment. The problem is usually formulated as an optimal control problem, and solved to provide the train drivers with detailed speed or control advice along the segment.[1]

The shape of the energy-efficient control/speed profile has been well studied with Pontryagin's maximum principle. The optimal train control strategy on level tracks with constant speed limit follows a sequence of (at most) four operations,

* Corresponding author.
E-mail address: R.Liu@its.leeds.ac.uk (R. Liu).

[1] It is possible that the suggested control advice is not able to perfectly reproduce the desired optimal speed profile in a practical train run due to, e.g., the inter-carriage interaction which is not considered in the optimal train control problem. It leads to the so-called train "cruise control" problem which deals with how to follow a prescribed speed profile. We do not consider the cruise control in this paper but refer interested readers to Li et al. (2014, 2015) for it.

http://dx.doi.org/10.1016/j.trc.2017.06.011
0968-090X/

which are maximum traction (MT), speedholding (SH), coasting (CS), and maximum braking (MB) (Asnis et al., 1985; Howlett, 1990; Ichikawa, 1968; Milroy, 1980). On undulating tracks with variable speed restrictions, these four operations can still compose the optimal control strategy (Albrecht et al., 2016a, 2016b; Howlett, 2000; Khmelnitsky, 2000; Liu and Golovitcher, 2003). State-of-the-art development on the optimal train control problem with continuous control is summarised by Albrecht et al. (2016a, 2016b), together with extension under more generalised assumptions. For trains with discrete control levels, the speedholding can be approximated by coast-power pairs (Cheng and Howlett, 1992, 1993; Cheng et al., 1999; Howlett, 1996; Howlett and Cheng, 1997; Pudney and Howlett, 1994). Comprehensive reviews of the analysis on classic single-train optimal control with continuous or discrete control can further be found in Scheepmaker et al. (2017) and Yang et al. (2016b).

Knowing the shape of the optimal control profile is not enough to drive the train in an energy-efficient way. The drivers need to be advised to take appropriate action at specific time or location of the journey, such as the speed to follow or the engine power to set. For this purpose, the optimal control problem has to be solved in order to generate the optimal control/speed profile along the journey (for reviews of the solution methods, see Scheepmaker et al., 2017; Yang et al., 2016b). Efficient algorithms have been developed based on the so-called indirect method, which tries to solve the Hamiltonian associated with the original optimal control problem based on Pontryagin's maximum principle (Albrecht et al., 2016a, 2016b; Howlett et al., 2009; Khmelnitsky, 2000; Liu and Golovitcher, 2003). Still, the indirect method cannot perfectly solve optimal train control in more complex settings, such as the optimal control of a fleet of trains, or the optimal train control with extra intermediate constraints applied on the train run, e.g., when the trains are required to dwell at the intermediate stations or to pass specific track locations within specific time windows. Alternatively, the optimal control formulations of these complex optimal train control problems can be first discretised by some numerical methods and then solved by mathematical programming techniques such as linear programming (Wang et al., 2013), nonlinear programing (Wang and Goverde, 2016a, 2016b, 2016c; Wang et al., 2013, 2014; Ye and Liu, 2016) and dynamic programming (Effati and Roohparvar, 2006; Franke et al., 2000; Ko et al., 2004; Vasak et al., 2009; Zhou et al., 2017). Quality of these discretisation-based methods depends on the discretisation step: a larger step-size requires less computation effort but yields larger energy cost and/or larger violation on the constraints. In addition, as one can find in Effati and Roohparvar (2006), Wang and Goverde (2016a) and Ye and Liu (2016), these discretisation-based methods would sometimes give fluctuating control/speed profiles which are difficult to follow or implement for the automatic train operation.

Besides the indirect methods, there are other methods built upon the pre-specified four operations. The driving plans based on coasting control (Açıkbaş and Söylemez, 2008; Chang and Sim, 1997; Wong and Ho, 2004) are easy to implement; however, their energy saving is restricted by the limited searching space and the pre-specified operation rules. Mathematical programming methods (Gu et al., 2014; Li and Lo, 2014a, 2014b; Yang et al., 2015) are developed in these years, which either require simplification on the train characteristics or track condition, such as constant maximum traction/braking force and/or zero running resistance, or rely on simulation to calculate the ordinary differential equations (ODEs) and integrals. Such simplification restricts the general application of these methods, for example, when the train speed is high such that the maximum traction/braking force is not able to maintain constant and the running resistance is not negligible; on the other hand, the simulation slows the solution process. Recently, Haahr et al. (2017) proposed a method based on dynamic programming. Forward and backward speed profiles were pre-generated under the four operations at prescribed discrete speed levels and at locations where the speed limit or gradient changes. Neither location nor time was discretised, so the fluctuating speed/control profiles were avoided. As only a limited number of discrete speed levels were considered, the prescribed journey time constraint could be slightly violated, and the minimum energy consumption obtained could be larger than the optimum when continuous speeds were considered.

In addition to the constraints at the two ends of a journey, the train control problem may sometimes involve intermediate constraints. For example, in Haahr et al. (2017) and Wang and Goverde (2016a, 2016c), the train needs to pass specific track locations within specific time/speed windows; in Ye and Liu (2016), the leading train is required to stop at the passing loop to let the following train overtake. The pseudospectral method (PM) has been used to solve these problems but it sometimes leads to undesired violent fluctuation on the control profiles (Wang and Goverde, 2016a; Ye and Liu, 2016). A dynamic-programming-based method recently developed by Haahr et al. (2017) appears to overcome such issue. The train control problems with intermediate constraints also include a large group of research on the energy-efficient subway line scheduling, where the train needs to run through the whole subway line while stopping at stations for passenger boarding and alighting (see Scheepmaker et al., 2017; Yang et al., 2016b for comprehensive reviews). To solve the subway line scheduling problems, previous literature usually assumed simplified train characteristics such as nil running resistance and constant maximum tractive/braking force, or relied on simulation/discretisation to calculate the speed, travel distance and energy consumption (Chevrier et al., 2013; Das Gupta et al., 2016; Li and Lo, 2014a, 2014b; Yang et al., 2015, 2016a; Yin et al., 2016; Zhou et al., 2017). Howlett (2016) recently pointed out that, when no speed restriction is imposed, the different interstation journeys should share a same optimal speed; however, there are still no effective methods to find either this optimal speed or the optimal interstation running times.

So far in this introductory part, we have been talking about the control of a single train. However, a train's optimal driving plan may be infeasible, or not optimal from the systematic point of view, when it is running close to other trains on the same track in the same direction. For instance, a following train may have to give up its optimal strategy in order to yield to the safe separation to the leading train; likewise, the leading train may also need to compromise its optimal driving plan for the advancing and energy saving of the following trains. Meanwhile, the trajectories of trains running in different directions can