ARTICLE IN PRESS

Transportation Research Part C xxx (2015) xxx–xxx

Contents lists available at ScienceDirect

Transportation Research Part C

journal homepage: www.elsevier.com/locate/trc

Ordered median hub location problems with capacity constraints

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ARTICLE INFO

Article history:
Received 12 May 2014
Received in revised form 24 March 2015
Accepted 13 May 2015
Available online xxxx

Keywords:
Hub location
Discrete ordered median problems
Capacities

ABSTRACT

The Single Allocation Ordered Median Hub Location problem is a recent hub model introduced in Puerto et al. (2011) that provides a unifying analysis of a wide class of hub location models. In this paper, we deal with the capacitated version of this problem, presenting two formulations as well as some preprocessing phases for fixing variables. In addition, a strengthening of one of these formulations is also studied through the use of some families of valid inequalities. A battery of test problems with data taken from the AP library are solved where it is shown that the running times have been significantly reduced with the improvements presented in the paper.

1. Introduction

Network design problems are among the most interesting models in combinatorial optimization. In the last years researchers have devoted a lot of attention to a particular member within this family, namely the hub location problem, that combines network design and location aspects of supply chain models, see the surveys (Alumur and Kara, 2008; Campbell et al., 2002; Campbell and O'Kelly, 2012). The main advantage of using hubs in distribution problems is that they allow to consolidate shipments in order to reduce transportation costs by applying economies of scale, which are naturally incorporated to the models through discount factors. Hub location problems have been studied from different perspectives giving rise to a number of papers considering alternative criteria to be optimized: the minimization of the overall transportation cost (sum) (see Campbell, 1996; Cánovas et al., 2007; Ernst and Krishnamoorthy, 1999; García et al., 2012; Labbé et al., 2005; Marín, 2005a,b; Marín et al., 2006), the minimization of the largest transportation cost or the coverage cost (Bollapragada et al., 2006; Campbell et al., 2007; Kara and Tansel, 2000; Kara and Tansel, 2003; Kratica and Stanimirovic, 2006; Meyer et al., 2009; Tan and Kara, 2007; Wagner, 2008), etc.

Apart from the choice of the optimization criterion, another crucial aspect in the literature on hub location, and in general in any location problem, is the existence or not of capacity constraints. One can recognize that although capacitated models are more realistic, the difficulty to solve them also increases in orders of magnitude with respect to their uncapacitated counterpart. In many cases new formulations are needed and a more specialized analysis is often required to solve even smaller sizes than those previously addressed for the uncapacitated versions of the problems. For this reason, capacitated versions of hub location problems have attracted the interest of locators in the last years, see (Aykin, 1994; Campbell, 1994; Contreras et al., 2009; Correia et al., 2010a,b; Ebery et al., 2000; Boland et al., 2004; Ernst and Krishnamoorthy, 1999; Marín, 2005a).

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http://dx.doi.org/10.1016/j.trc.2015.05.012
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Along the same line, it is worth mentioning some other references that assume congestion at hubs, as congestion acts as a limit on capacity, see (De Camargo et al., 2009; Elhedhli and Hu, 2004; Marianov and Serra, 2003).

An interesting version of hub location model is the Capacitated Hub Location Problem with Single Allocation (CSA-HLP), see (Contreras et al., 2009; Correia et al., 2010a; Ernst and Krishnamoorthy, 1999). In this context, single allocation means that incoming and outgoing flow of each site must be shipped via the same hub. In contrast to single allocation models, where binary variables are required in the allocation phase, multiple allocation allows different delivery patterns which in turns implies the use of continuous variables, simplifying the problems. The CSA-HLP model incorporates capacity constraints on the incoming flow at the hubs coming from origin sites or even simpler, on the number of non-hub nodes assigned to each hub. The inclusion of capacity constraints makes these models challenging from a theoretical point of view. Regarding its applicability we cite one example described in Ernst and Krishnamoorthy (1999) based on a postal delivery application, where a set of n postal districts (corresponding to postcode districts represented by nodes) exchange daily mail. The mail between all the pairs of nodes must be routed via one or at most two mail consolidation centers (hubs). In order to meet time constraints, only a limited amount of mail could be sorted at each sorting center (mail is just sorted once, when it arrives to the first hub from origin sites). Hence, there are capacity restrictions on the incoming mail that must be sorted. The problem requires to choose the number and location of hubs, as well as to determine the distribution pattern of the mail.

The CSA-HLP has received less attention in the literature than its uncapacitated counterpart. Campbell (1994) presented the first Integer Mathematical Programming formulation for the Capacitated Hub Location Problem. This formulation was strengthened by Skorin-Kapov et al. (1996). Ernst and Krishnamoorthy (1999), proposed a new model involving three-index continuous variables and developed a solution approach based on Simulated Annealing where the bounds obtained were embedded in a branch-and-bound procedure devised for solving the problem optimally. Recently, Correia et al. (2010a) have shown that this formulation may be incomplete and they propose an additional set of inequalities to assure the validity of the model in all situations. A new formulation using only two-index variables was proposed by Labbé et al. (2005), where a polyhedral analysis and new valid inequalities were addressed. Although this formulation had only a quadratic number of variables, the number of constraints was exponential, and to solve it, the authors developed a branch-and-cut algorithm based on their polyhedral analysis. Contreras et al. (2009) presented for the same problem a Lagrangian relaxation enhanced with reduction tests that allowed the computation of tight upper and lower bounds for a large set of instances.

In two recent papers, (Puerto et al., 2011, 2013), a new model of hub location, namely the Single Allocation Ordered Median Hub Location problem (SA-OMHLP), has been introduced and analyzed. This problem can be seen as a powerful tool from a modeling point of view since it allows a common framework to represent many of the previously considered criteria in the literature of hub location. Moreover, this approach is a natural way to represent the differentiation of the roles played by the different parties (origins, hubs and destinations) in logistics networks (Fonseca et al., 2010, Kalcsics et al., 2010a,b; Marín et al., 2009). This model does not assume, in advance, any particular structure on the network (Contreras et al., 2009, 2011). Instead of that this structure is derived from the choice of the parameters defining the objective function. Apart from the above mentioned characteristics, ordered median objectives are also useful to obtain robust solutions in hub problems by applying k-centrum, trimmed-mean or anti-trimmed-mean criteria. It is worth mentioning that although it is called single allocation, its meaning slightly differs from the classical interpretation in hub location where each site is allocated to just one hub and all the incoming and outgoing flow to-from this site is shipped via the same link (the one joining this site and its allocated hub). In this model, single allocation means that all the outgoing flow is delivered through the same hub, but the incoming flow can come from different hubs. Actually, this is a mixed model and basically the same situation described above, about postal deliveries, naturally fits in this framework assuming that letters from the same origin should be sorted, with respect to their destinations, in the same place and from there they are delivered via their cheapest routes. Observe that in this scheme it is also natural that incoming flow in a final destination comes from different hubs.

The SA-OMHLP distinguishes among segmented origin–destination deliveries giving different scaling factors to the origin–hub, hub–hub and hub–destination links. The cost of each origin–first hub link is scaled by a factor that depends on the position of this cost in the ordered sequence of costs from each origin to its corresponding first hub (Boland et al., 2006; Marín et al., 2009; Nickel and Puerto, 2005). Moreover, the overall interhub cost and hub-destination cost are multiplied by different economies of scale factors. The goal is to minimize the overall shipping cost under the above weighting scheme. The reader may note that the first type of scaling factors mentioned above adds a "sorting" problem to the underlying hub location model, making its formulation and solution much more challenging. This model and two different formulations were introduced in Puerto et al. (2013) while a specialized B&B&Cut algorithm was developed in Puerto et al. (2011) and Ramos (2012). None of those formulations could handle capacities since the computation burden of the problems was very high demanding. Thus, the SA-OMHLP with capacity constraints, i.e. Capacitated Single Allocation Ordered Median Hub Location problem (CSA-OMHLP) is currently an open line of research.

In this paper, we analyze in depth the CSA-OMHLP trying to obtain a better knowledge and alternative ways to solve it. Thus, the contributions of this paper are threefold. First, it combines for the first time three challenging elements in location analysis: hub facilities, capacities and ordered median objectives, proposing a promising IP formulation which remarkably reduces the number of decision variables. Second, we develop new theoretical results which provide additional knowledge on the structure of the solutions set and allow to design an efficient cut and branch approach to solve the problem. Indeed, we are giving a better polyhedral description of the considered problem (fourteen families of valid inequalities). In addition to this strengthening, we also develop two new preprocessing phases which are shown to be very effective in solving the