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Optimization of horizontal alignment geometry in road design and reconstruction

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ABSTRACT

This paper presents a general formulation for optimization of horizontal road alignment, composed of tangential segments and circular curves suitably connected with transition curves (clothoids). It consists of a constrained optimization problem where the objective function is given by a line integral along the layout. The integrand is a function representing the cost of the road going through each point and, by considering different costs, a wide range of problems can be included in this formulation. To show it, we apply this methodology to three different situations. The two first cases are related with the design of a new road layout and used to solve a pair of academic examples. The third problem deals with the improvement of a road adapting the old path to current legislation, and it is solved taking as case study the reconstruction project for a regional road (NA-601) in the north of Spain.

1. Introduction

The challenge of achieving the *optimal* alignment for a road design is a complex problem and highly topical in Civil Engineering. In general, the aim is to obtain an *admissible* layout minimizing the final cost of the working execution. The layout must also be admissible and fit the constraints given by the legislation of each country as well as the inherent characteristics of the location, for example, regions where the road must go through or restricted areas.

Mathematical modeling and optimization techniques can be a powerful tool seeking optimal alignment. Nowadays the bibliographic references are extensive in that topic. Papers can be classified according to different aspects. With respect to the target for optimizing, there are papers devoted to horizontal alignment only (Jong et al., 2000; Easa and Mehmood, 2008; Lee et al., 2009; Mondal et al., 2015; Pushak et al., 2016), to vertical alignment only (Jong and Schonfeld, 2003; Hare et al., 2014, 2015) and to the three dimensional alignment (Chew et al., 1989; Jong and Schonfeld, 2003; Maji and Jha, 2009; Kang et al., 2012; Yang et al., 2014; Hirpa et al., 2016). With respect to the cost, some papers work with a specific objective, as traffic safety (Easa and Mehmood, 2008), length (Lee et al., 2009) or earthwork (Hare et al., 2014, 2015; Mondal et al., 2015), while others deal with costs including elements depending on the location (land acquisition, environmental cost, terrain, etc.), length (pavement, maintenance, ...), traffic safety (visibility, secure overtaking, etc.), and others (Chew et al., 1989; Jong and Schonfeld, 2003; Maji and Jha, 2009; Kang et al., 2012; Yang et al., 2014; Hirpa et al., 2016; Li et al., 2016). These last papers also can be sub-classified from the formulation of the optimization problem into single-objective (Chew et al., 1989; Jong and Schonfeld, 2003; Kang et al., 2012) and multi-objective optimization problems (Maji and Jha,

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2009; Yang et al., 2014; Hirpa et al., 2016). In any case, it should be noted that earthwork cost is one of the most important economic costs. Most of the papers dealing with alignment optimization work with earthwork cost and it is also studied in many other optimization papers (Easa, 1988; Hare et al., 2011; de Lima et al., 2013; Burdett and Kozan, 2014). Finally, the optimization approach is also a very important task in this field and, for example, in Kang et al. (2012), and more recently in Li et al. (2016), a classification of the papers according to this aspect can be found.

In what concerns to the alignment geometry, many different models have been used in the literature. A horizontal road alignment should be a series of straights (tangents) and circular curves jointed by means of transition curves. Although at present time there exist different approaches (Kobryń, 2014), the best alternative for transition curves are clothoids whose equations are complex and hamper to obtain an explicit parametrization of the path. To avoid that problem, Chew et al. (1989) select a polynomial type equation to adjust the road design. Lee et al. (2009) work with a polygonal path, and only once the optimization process has ended, curves (clothoid-circular-clothoid) are built over the optimal polygonal route. In Jong et al. (2000), Mondal et al. (2015) and Hirpa et al. (2016) an alignment consisting of straights and circular curves are constructed in terms of their respective decision variables, and in Kang et al. (2012) transition curves (clothoids) are incorporated to the horizontal alignment, but an explicit parametrization of the path is not given.

In this work we deal with the horizontal road design considering that the layouts are composed by tangents and circular curves properly connected with clothoids. The aim is to give a simple and general formulation of an optimization problem to obtain the horizontal road alignment. With this goal in mind, we begin (Section 2) by determining the decision variables of the problem and, taking as starting point a previous work (Vázquez-Méndez and Casal, 2016), we detail an algorithm to compute a complete parametrization of the alignment from those variables. Afterwards, in Section 3, we introduce a standard approach to compute optimal layouts, based on framing the horizontal alignment design as a constrained optimization problem. The main idea lies in thinking that every geographical point has a *price* and that the objective function is the sum of all those *prices* creating the road. A wide interpretation of *price* allows us to include in this framework a large amount of diverse problems. To illustrate this fact we employ the proposed methodology to three different cases. In the first two (Section 4) we look for the shortest path joining two terminals, avoiding certain obstacles (protected areas to preserve or either zones to dodge due to high slopes). In both cases, we show the effectiveness of our formulation by solving the corresponding problems in simple academic examples. The third problem (Section 5) is related with a road reconstruction project (particularly, with improving alignment to adapt it to current legislation). In this case, we take as case study the road reconstruction project of a regional road (NA-601) in Navarra, northern Spain, and we compared our results with the real performed project. Finally (Section 6), some brief and interesting conclusions are sketched.

2. Mathematical model for horizontal alignment

2.1. Design variables

The problem arising is the design of a road between two given points $a, b \in \mathbb{R}^2$. The horizontal road alignment to be implemented should be formed by the suitable combination of straight sections, circular curves and transition curves, which in our model will be clothoids. If the path consists of $N+1$ tangents, it is unequivocally determined by the vertices ($v_i = (x_i, y_i),\ i = 1, \ldots, N$) where these tangents intersect, and the radii ($R_i \geqslant 0,\ i = 1, \ldots, N$) and angles ($\omega_i \geqslant 0,\ i = 1, \ldots, N$) of the circular curves (see Fig. 1). Thus, for each $N \in \mathbb{N}$ we define

$$\mathbf{x}^N = (x_1, y_1, R_1, \omega_1, \ldots, x_N, y_N, R_N, \omega_N) \in \mathbb{R}^{4N},$$

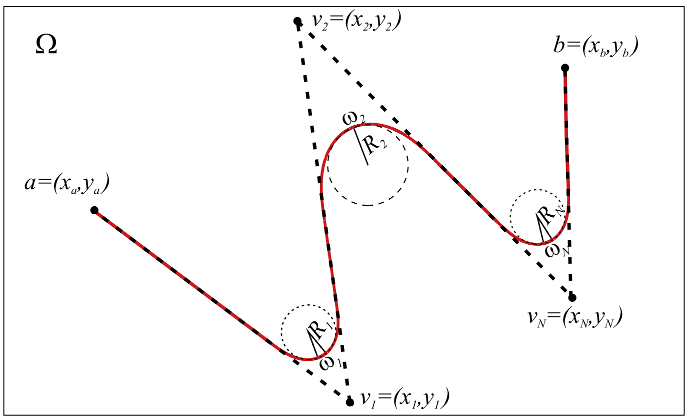

Fig. 1. Horizontal road alignment connecting two terminals a and b. The decision variables (components of vector $\mathbf{x}^N$) are shown. The constructed path ($C_{\mathbf{x}^N}$) is plotted and the tangents are also plotted in dashed line.