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Consensus, dissension and precision in group decision making by means of an algebraic extension of hesitant fuzzy linguistic term sets

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ABSTRACT

Present measures of the degree of agreement in group decision-making using hesitant fuzzy linguistic term sets allow consensus or agreement measurement when decision makers' assessments involve hesitance. Yet they do not discriminate with different degrees of consensus among situations with discordant or polarized assessments. The visualization of differences among groups for which there is no agreement but different possible levels of disagreement is an important issue in collective decision-making situations. In this paper, we propose new collective and individual consensus measures that explicitly consider the hesitance of the decision makers' hesitance in giving an opinion and also the gap between non-overlapping assessments, thus allowing the measurement of the polarization present within the group's opinions. In addition, an expert's profile is defined by considering the expert's behavior in previous assessments in group decision-making processes in terms of precision and dissension.

Introduction

Several studies have shown that, in general, people do not use purely quantitative models when expressing preferences and interests and are more comfortable using global or abstract forms, that can be understood as models based on qualitative or linguistic information [1–3]. Analogously, in Group Decision-Making (GDM) environments, the design of systems to facilitate decision-making processes is considered suitable for describing alternatives to be made in terms of non-numerical values and reflect the uncertainty inherent in human reasoning [4–8]. In the literature, this impreciseness has been modeled with intervals or fuzzy values through a linguistic approach [9–11].

Rodríguez et al. [9] introduced the Hesitant Fuzzy Linguistic Term Sets (HFLTSs) over a well-ordered set of linguistic labels to deal with decision-making situations through hesitant fuzzy linguistic assessments. In this way, one can express not only the uncertainty but also the hesitance inherent in human reasoning. There are several contributions in the literature that have studied HFLTSs, their properties, aggregation functions, preference relations, distances and so on [12–16]. These approaches have contributed either from a theoretical point of view or by proposing different applications. An algebraic extension of the set of HFLTSs is presented in [17] to take into account the gap between non-overlapping assessments.

In recent times, consensus in GDM problems through HFLTSs has been studied by several approaches [12,18–22]. While some of them focus on the aim of quantifying the level of agreement, some others focus on the consensus reaching process. The problem is set, for all of them, with a group of experts or Decision Makers (DMs) evaluating a set of several alternatives by means of HFLTSs. Despite this, some differences emerge among the approaches that try to quantify the consensus level. A first key difference between them is that, while some approaches study, for each alternative, the consensus of an expert with respect to the rest of the group [12,20], others study the consensus of the whole group on each alternative [18,19,21]. Both types of consensus approaches might be useful under different kinds of situations: while approaches of the first type can be used to evaluate the relation of each expert with respect to the group, approaches of the second type can be used to evaluate the available alternatives. For instance, when in a GDM process the most dissenting decision makers are asked to reconsider their opinion, a measure of the first kind should be used. On the contrary, when everyone is asked to reconsider his or her assessment on the most controversial alternative, a second type measure should be used instead. In this paper, we propose a new measure of consensus that can be adapted to the measurement of both individual and collective consensus.

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The second main difference among approaches lies in whether the definition of the measure of consensus is based on the concept of distance or on the concept of similarity. On the one hand, the consensus level presented in [12] is a distance-based measure. According to the distance that it is used in [12], if two opinions do not overlap, the consensus level is always zero, regardless how far apart the opinions are. This is because the distance used does not take into consideration the gap between HFLTSs in the cases in which the intersection is the empty set. In this paper we define more accurate agreement measures, based on the distance presented in [13] that does take into consideration this gap. On the other hand, the measures presented in [18–21] are not distance-based but similarity-based. The concept of similarity between HFLTSs is presented in [18], and later used in [21], based on the comparison, between two experts, of their preferences of a given alternative over another one and extended in [19] as a comparison, between two experts, of their assessment of a specific alternative. In any case, this similarity concept neither takes into consideration how distant non-overlapping assessments are nor the level of hesitance used by the experts when assessing an alternative. The measures presented in this paper solve these issues by considering both the hesitance of the assessments and the gap between them if they do not overlap.

Selecting or prioritizing suitable experts or DMs is a frequent problem in GDM applications in real situations [23,24]. This paper introduces the concepts of preciseness and dissent of an expert assessing a set of alternatives. This allows the definition of an expert's profile, which keeps track of how experts have made his/her previous assessments with respect to how precise or how dissenting they are. These profiles characterize the up-to-date behavior of experts in GDM processes and can be useful for the task of selecting the appropriate experts to form part of future committees or decision groups.

The rest of this paper is structured as follows: first, Section 1 presents a summary of the basic concepts in the literature that are used throughout the paper. A new degree of consensus for the whole group on each alternative is introduced in Section 2 with a further comparison study with other similar measures. Section 3 defines a different degree of consensus for an expert with respect to the group and it is also compared with the similar existing measures. A precision–dissension profile is presented in Section 4 to keep track of the assessments of a DM within several groups. Finally, Section 5 presents the main conclusion and lines of future research.

1. Theoretical framework

The aim of this section is to provide a summary of basic concepts related to HFLTSs that appear throughout this paper. In particular, a special focus on the distance between HFLTSs that is used in this work is required.

From this point onwards, let $\mathcal{S}$ denote a finite total ordered set of linguistic terms, $\mathcal{S} = \{a_1, \dots, a_n\}$ with $a_1 < \dots < a_n$.

Definition 1 [9]**.** A hesitant fuzzy linguistic term set (HFLTS) over $\mathcal{S}$ is a subset of consecutive linguistic terms of $\mathcal{S}$, i.e., $\{x \in \mathcal{S} \mid a_i \leq x \leq a_j\}$, for some $i, j \in \{1, \dots, n\}$ with $i \leq j$.

Following the concept of uncertain linguistic term introduced by Xu [25], in this paper we denote HFLTSs by linguistic intervals. Thus, for the rest of this article, the HFLTS $\{x \in \mathcal{S} \mid a_i \leq x \leq a_j\}$ is denoted as $[a_i, a_j]$ or, if $j = i$, $\{a_i\}$. In addition, $\mathcal{H}_\mathcal{S}$ represents the set of all the possible HFLTSs over $\mathcal{S}$ including the empty HFLTS, $\emptyset$.

In order to define a suitable distance between two HFLTSs that takes into consideration not just the intersection of them, but also the gap between them if they do not intersect, an algebraic extension of the set $\mathcal{H}_\mathcal{S}^* = \mathcal{H}_\mathcal{S} - \{\emptyset\}$ is presented in [17] as $\overline{\mathcal{H}_\mathcal{S}}$ different than the extension presented in [14] that includes HFLTS with non-consecutive linguistic terms from $\mathcal{S}$. This algebraic extension includes the concepts of the *negative HFLTSs*, $-\mathcal{H}_\mathcal{S}^* = \{-H | H \in \mathcal{H}_\mathcal{S}^*\}$, the *zero HFLTSs*, $\mathcal{A} = \{\alpha_0, \dots, \alpha_n\}$ and the *positive HFLTSs*, $\mathcal{H}_\mathcal{S}^*$. The graph of this set is presented in Fig. 1.

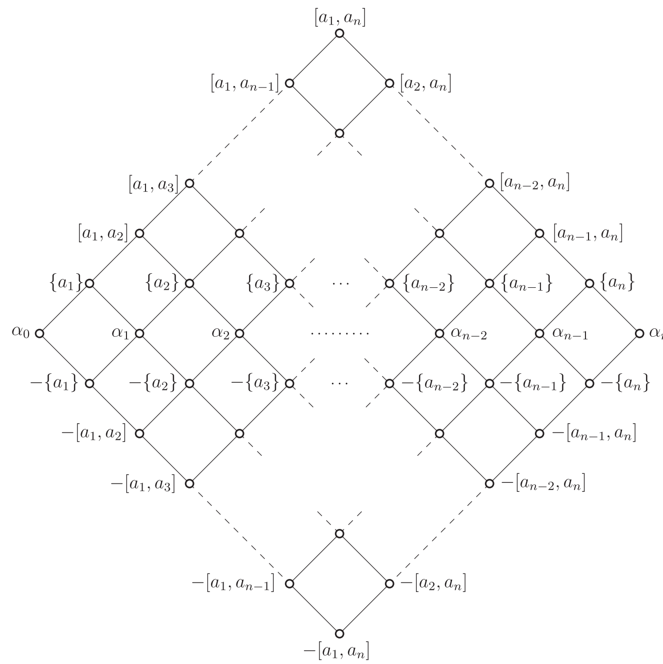

Fig. 1. Graph of the extended set of HFLTSs, $\overline{\mathcal{H}_\mathcal{S}}$.

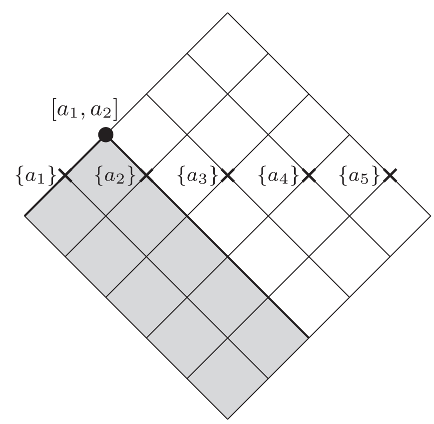

Fig. 2. Elements of $\overline{\mathcal{H}_\mathcal{S}}$ included in $[a_1, a_2]$.

In the frame of $\overline{\mathcal{H}_\mathcal{S}}$, an *extended inclusion relation* is introduced based on the graph of $\overline{\mathcal{H}_\mathcal{S}}$ (Fig. 1) and the usual inclusion relation between HFLTSs. Fig. 2 shows, as an example, all the elements of $\overline{\mathcal{H}_\mathcal{S}}$ included in $[a_1, a_2]$ according to the extended inclusion relation. Additionally, this extended inclusion relation is used to extend the connected union and the intersection of HFLTSs to an operation between elements of $\overline{\mathcal{H}_\mathcal{S}}$.

Definition 2 [17]**.** Given $H_1, H_2 \in \overline{\mathcal{H}_\mathcal{S}}$, then:

a) The extended connected union of H_1 and H_2, $H_1 \sqcup H_2$, is defined as the least element that contains H_1 and H_2, according to the extended inclusion relation.
b) The extended intersection of H_1 and H_2, $H_1 \sqcap H_2$, is defined as the largest element being contained in H_1 and H_2, according to the extended inclusion relation.