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Fusing time, frequency and shape-related information: Introduction to the Discrete Shapelet Transform's second generation (DST-II)

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A B S T R A C T

This article introduces the second generation of the Discrete Shapelet Transform (DST-II), a tool created for fusing three types of information: time, frequency and shape-related. Considered a particular Discrete Wavelet Transform (DWT), it allows a productive time-frequency-shape (TFS) joint analysis. In the proposed approach, both the procedure to attain the corresponding filters coefficients and the interpretation of the transformed signal are simplified in relation to the usage of its predecessor, i.e., the DST. Throughout the article, the DST-II formulation is described in detail, including a numerical example, a prototype for use in a diversity of fields and an application on spike and overlap sorting, reassuring the efficacy of the new transform.

1. Introduction

Since Alfréd Haar, the father of wavelets, introduced fundamental concepts in 1909 [1] and Ingrid Daubechies, the mother of wavelets, brightly consolidated them decades later [2], the Discrete Wavelet Transform (DWT) [3] has been placed at the forefront of signal analysis, fusing two types of information: temporal and spectral. Through the years, many other scientists have published their relevant contributions to the field of wavelet theory. Palghat Vaidyanathan, Martin Vetterli, Stéphane Mallat, David Donoho, Gilbert Strang and others are examples of unforgettable names, just to mention a few. Additionally, our information fusion community has continuously presented DWT-based and DWT-inspired advances, as shown in [4–13].

In order to further evolve ordinary wavelet analysis, the Discrete Shapelet Transform (DST) was recently created and presented to the scientific community, as documented in [14], in a previous cooperative work of mine. Exactly as the Discrete Wavelet Transform (DWT) does [15], the DST allows the time-support of frequencies to be found, however, with a special advantage: concomitantly, it quantifies the degree of similarity between the signal under analysis and a pre-specified shape. Its work principle consists of a fractal-based criterion [16] used to redefine the original Daubechies' DWTs in such a way that a time-frequency-shape (TFS) joint analysis is performed. Thus, the DST fuses three types of information: temporal, spectral and shape-related.

On one hand, the original DST formulation demonstrated that TFS joint analysis is feasible. On the other, my objective this time is to improve that technique by defining the second generation of the transform, i.e., DST-II. Particularly, the new tool replaces the fractal-based criterion used for shape matching by a correlation-based formulation, favouring the solution of the non-linear system of equations that produces the filters coefficients and allowing a simplified interpretation of the transformed signal. Thus, the DST-II is better than its predecessor for joint TFS analysis, stimulating its usage in a diversity of fields.

In suggesting possible future trends for the scientific community, this paper is organised as follows. Supported by a short review on DSTs and the original Daubechies' DWT, presented in Section 2, Section 3 introduces the DST-II and its inverse (IDST-II). Proceeding, Section 4 shows a numerical example, while Section 5 describes the tests and results obtained during the analysis of simulated and biological data and, lastly, Section 6 reports the conclusions that are followed by the references. Readers of this article are strongly encouraged to learn my previous piece on the original DST [14] before proceeding any further.

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2. A short review

2.1. DSTs: the first generation

A deep review on DSTs is superfluous due to the detailed description presented previously in [14]. However, there are a few important points to be mentioned regarding the elements associated with that transform. Similarly to the ordinary DWT, they are:

- $p[\cdot]$ and $q[\cdot]$, so that $q_k = (-1)^k p_{N-k-1}$, form the quadrature mirror filter (QMF) [3] pair of finite impulse response (FIR) filters [17] with support-size $N \geq 4$ used for signal analysis, being N even. They present, respectively, low-pass and high-pass frequency responses with not necessarily linear phases [17]. Accordingly, these are the filters used in conjunction with Mallat's algorithm [18] to obtain the transformed signal from the input, exactly as in the original DWTs for which they are usually known as $h[\cdot]$ and $g[\cdot]$;
- $\bar{p}[\cdot]$, so that $\bar{p}_k = p_{N-k-1}$, and $\bar{q}[\cdot]$, so that $\bar{q}_k = (-1)^{k+1} p_k$, form the pair of filters used for signal re-synthesis. In the scope of the DWT, they would be respectively known as $\bar{h}[\cdot]$ and $\bar{g}[\cdot]$;
- $\Gamma(x) = \sum_k p_k \Gamma(2N - k)$ and $\Theta(x) = \sum_k q_k \Gamma(2N - k)$, respectively known as *major shapelet* and *minor shapelet*, correspond to scaling and wavelet functions of the DWT [3], i.e., $\Phi(x)$ and $\Psi(x)$;
- the conditions $\bar{P}[z] = Q[-z]$, $\bar{Q}[z] = -P[-z]$ and $\bar{P}[z]P[z] + \bar{Q}[z]Q[z] = 2z^{-N+1}$, all in Z domain [17], imply that $p[\cdot]$, $q[\cdot]$, $\bar{p}[\cdot]$ and $\bar{q}[\cdot]$ form a perfect-reconstruction filter bank (PRFB) [3].

Particularly, the procedure to obtain the DST filter $q[\cdot]$ is the same used to generate the Daubechies' filter $g[\cdot]$, as reviewed ahead, albeit with one difference: the former formulation replaces one vanishing moment condition from the latter by a fractal-based matching equation. The DST-II, however, is based on a different approach.

Complementarily, it is important to recall that the DST($s[\cdot]$) preserves the length of the input signal $s[\cdot]$, hereafter referred to as X, that is a power of 2. Furthermore, X allows for the decomposition until level $j = \left(\frac{\log(X)}{\log(2)}\right)$. Once $s[\cdot]$ is decomposed, two other signals with lengths $\frac{X}{2}$ are produced: *master* and *second-rated*. The former and the latter result, respectively, from the convolution of $s[\cdot]$ with $p[\cdot]$ and the convolution of $s[\cdot]$ with $q[\cdot]$, both followed by a downsampling by 2 and a wrap-around procedure [18]. Lastly, the concatenation of *master* with *second-rated* characterizes the DST. From the former, the decomposition can continue recursively until reaching the highest possible level. DST-II inherits all the terminology and decomposition procedures from DST.

2.2. The discrete Daubechies' transform

There are distinct ways to explain how the Daubechies's wavelets [3] were constructed. Particularly, that of my current interest, which first produces the high-pass filter, i.e., $g[\cdot]$ with support-size N, and then generates the other elements, i.e., $h[\cdot]$, $\bar{h}[\cdot]$, $\bar{g}[\cdot]$, $\Phi(x)$ and $\Psi(x)$, based on it, will be reviewed here. The specific procedure is:

- STEP Daub$_1$: Force $g[\cdot]$ to have unitary energy so that the DWT preserves that of the input signal, i.e.,

$$\sum_{k=0}^{N-1} {g_k}^2 = 1. \tag{1}$$

  This condition is equivalent to others, as $\sum_{k=0}^{N-1} h_k = \sqrt{2}$, implying that the scaling function has one non-vanishing moment;
- STEP Daub$_2$: Impose $\frac{N}{2}$ vanishing moments on the wavelet function, i.e.,

$$\sum_{k=0}^{N-1} g_k k^b = 0 \quad , \tag{2}$$

  for $b = 0, 1, \ldots, \frac{N}{2} - 1$;
- STEP Daub$_3$: Define $\frac{N}{2} - 1$ orthogonality conditions related to the translations of the filter so that the transformation matrix used to carry out Mallat's algorithm [18] is orthogonal, allowing signal re-synthesis based on its transpose:

$$\sum_{k=0}^{N-1} g_k g_{k+2l} = \delta_{0,l}, \tag{3}$$

  being δ the Dirac delta and $l \in \mathcal{Z}$;
- STEP Daub$_4$: Group together the only equation of step Daub$_1$, the $\frac{N}{2}$ equations of step Daub$_2$ and the $\frac{N}{2} - 1$ equations of step Daub$_3$, resulting in a non-linear system of N equations in N unknowns. Then, solve the system using any iterative numerical procedure, such as Gauss-Siedel, Jacobi or Newton's methods [20], to obtain the high-pass filter $g[\cdot]$.
- STEP Daub$_5$: Obtain the filter $h[\cdot]$ so that $h_k = (-1)^{k+1} g_{N-k-1}$ in order to complete the analysis filter pair. If the inverse DWT (IDWT) is required, obtain the filters $\bar{h}[\cdot]$ and $\bar{g}[\cdot]$, so that $\bar{h}_k = h_{N-k-1}$ and $\bar{g}_k = (-1)^{k+1} h_k$, characterizing the re-synthesis filter pair. Lastly, in order to discover the shapes of orthonormal basis associated with the analysis filter pair, as explained in [21], obtain the scaling function $\Phi(x) = \sum_k h_k \Phi(2N - k)$ and the wavelet function $\Psi(x) = \sum_k g_k \Phi(2N - k)$.

3. The DST-II

3.1. DST-II definition and formulation

The components of the DST and the DST-II are just the same, i.e., $p[\cdot]$, $q[\cdot]$, $\bar{p}[\cdot]$, $\bar{q}[\cdot]$, $\Gamma(x)$ and $\Theta(x)$. The easiest way to calculate them is to obtain, first, the analysis filter $q[\cdot]$ for which a few restrictions apply regarding the DST-II:

1. the filter support-size is $N \geq 6$. This is required due to the fact that the DST-II has $\frac{N}{2} - 2$ vanishing moments in its minor shapelet, equivalently to the wavelet function, as detailed ahead. Therefore, $N < 6$ would produce no vanishing moment in such function, disturbing the proposed transform;
2. the filter support-size is necessarily even, as in the DWT theory [3], otherwise a perfect-reconstruction can not be achieved;
3. the signal to be matched, $m[\cdot]$, representing the pattern to be identified by the DST-II, has necessarily an odd size equal to $N + 1$.

The particular procedure to determine $q[\cdot]$ is:

- STEP Shp$_1$: Force the filter to have unitary energy so that the DST-II preserves that of the input signal, i.e.,

$$\sum_{k=0}^{N-1} {q_k}^2 = 1. \tag{4}$$

- STEP Shp$_2$: Impose $\frac{N}{2} - 2$ vanishing moments for the major shapelet function, i.e.,

$$\sum_{k=0}^{N-1} q_k k^b = 0, \tag{5}$$

  for $b = 0, 1, \ldots, \frac{N}{2} - 3$;