Information Fusion 38 (2017) 22–34

Contents lists available at ScienceDirect

Information Fusion

journal homepage: www.elsevier.com/locate/inffus

Double hierarchy hesitant fuzzy linguistic term set and MULTIMOORA method: A case of study to evaluate the implementation status of haze controlling measures

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ARTICLE INFO

Article history:
Received 28 January 2017
Revised 9 February 2017
Accepted 20 February 2017
Available online 1 March 2017

Keywords:
Multiple criteria decision making
Double hierarchy linguistic term sets
Double hierarchy hesitant fuzzy linguistic term sets
MULTIMOORA method
Haze controlling measures

ABSTRACT

In recent years, hesitant fuzzy linguistic term sets (HFLTSs) have been studied by many scholars and are becoming gradually mature. However, some shortcomings of HFLTS also emerged. To describe the complex linguistic terms or linguistic term sets more accurately and reasonably, in this paper, we introduce the novel concepts named double hierarchy linguistic term set (DHLTS) and double hierarchy hesitant fuzzy linguistic term set (DHHFLTS). The operational laws and properties of the DHHFLTSs are developed as well.

Afterwards, we investigate the multiple criteria decision making model with double hierarchy hesitant fuzzy linguistic information. We develop a double hierarchy hesitant fuzzy linguistic MULTIMOORA (DHHFL-MULTIMOORA) method to solve it. Furthermore, we apply the DHHFL-MULTIMOORA method to deal with a practical case about selecting the optimal city in China by evaluating the implementation status of haze controlling measures. Some comparisons between the DHHFL-MULTIMOORA method and the hesitant fuzzy linguistic TOPSIS method are provided to show the advantages of the proposed method.

1. Introduction

Hesitant fuzzy linguistic term set (HFLTS), combined by hesitant fuzzy set (HFS) [1–4] and fuzzy linguistic approach [5], was developed by Rodríguez et al. [6] in 2012. It is a useful tool to deal with qualitative information given that the HFLTS can represent the linguistic information that is much more in line with people's cognitions and expressions. In recent years, amounts of scholars have researched the HFLTS theory from different research directions including information aggregation [7–9], fuzzy measures [10–14], preference relations [13,15–17], decision making [12–14,18–23], etc.

As the researches on HFLTSs have been studied in-depth and the HFLTS theory is becoming gradually mature, some shortcomings of HFLTSs, however, also emerged from two aspects:

a) In group decision making process, the aggregated hesitant fuzzy linguistic information cannot represent the important degree or the frequency of each linguistic term included in the HFLTS.
b) The HFLTS is not accurate enough to describe some more complex linguistic terms or linguistic term sets (LTSs).

For the first shortcoming, Pang et al. [24] defined a probability linguistic term set (PLTS) to generalize the HFLTSs by adding the probability information of each single linguistic term, which is a very reasonable method for saving all original linguistic information given by the experts in group decision making process. Furthermore, by utilizing the PLTSs, the experts can not only provide several linguistic evaluation values over an object (alternative or criterion), but also reflect the probability information of each element included in the LTS. Later, some scholars have studied the PLTSs from different aspects, among others: probabilistic linguistic preference relation and consistency measures [25], probabilistic linguistic vector-term sets to promote the application of multi-granular linguistic information [26], comparative procedure-based multiple criteria decision making (MCDM) problems [27], novel operational laws of PLTSs based on two equivalent transformation functions [28].

For the second shortcoming, it is obvious that sometimes the HFLTS cannot describe some complex linguistic terms or LTSs ac-

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http://dx.doi.org/10.1016/j.inffus.2017.02.008
1566-2535/

curately. For example, let $S = \{s_{-3} = none, s_{-2} = very\ low, s_{-1} = low, s_0 = medium, s_1 = high, s_2 = very\ high, s_3 = perfect\}$ be a LTS, then we can utilize $\{s_2, s_3\}$, $\{s_{-1}, s_0, s_1\}$ and $\{s_2\}$ to express the linguistic expressions "*more than very high*", "*between low and high*" and "*very high*". However, sometimes, we may need to use some more complex linguistic terms to represent our comprehensive opinions such that "*entirely high*", "*just right medium*", "*a little high*", etc. Considering that we cannot use any method or theory to solve this problem, in this paper, we introduce a novel concept: double hierarchy linguistic term set (DHLTS). Generally, the DHLTS consists of two hierarchy LTSs (denoted by the first hierarchy LTS and the second hierarchy LTS). The second hierarchy LTS is a linguistic feature or detailed supplementary of each linguistic term included in the first hierarchy LTS. Let the above LTS S be the first hierarchy LTS, and $O = \{o_{-3} = far\ from, o_{-2} = only\ a\ little, o_{-1} = a\ little, o_0 = just\ right, o_1 = much, o_2 = very\ much, o_3 = entirely\}$ be the second hierarchy LTS. Then we can describe "*entirely high*", "*just right medium*", "*a little high*" with DHLEs (the element included in the DHLTS), which are denoted as $s_{1<o_3>}$, $s_{0<o_0>}$ and $s_{1<o_{-1}>}$, respectively. Based on the DHLTS, we can develop a double hierarchy hesitant fuzzy linguistic term set (DHHFLTS). The DHHFLTS is a novel concept, which can be used to deal with some practical MCDM problems with linguistic information.

MCDM is one of the most important branches in decision analysis theory and many fruitful results and models have been achieved related to this area. Among the widely used MCDM methodologies, the multiple multi-objective optimization by ratio analysis (MULTIMOORA) method and its extensions have been investigated by many scholars [29–40]. As an effective and comprehensive method, it combines three aspects including the ratio system, the reference point, and the full multiplicative form. The MULTIMOORA method and its extended forms have been applied to many fields such as transition economies [29], human resource management and performance management [30], EU Member States updating management [31], heating losses ranking in a building [32], supplier selection [34] and so on.

In this paper, we mainly develop a double hierarchy hesitant fuzzy linguistic MULTIMOORA (DHHFL-MULTIMOORA) method to deal with practical MCDM problems. We apply the DHHFL- MULTIMOORA method to a case of selecting the best city in China by evaluating the implementation status of haze controlling measures. Some comparisons between the DHHFL-MULTIMOORA method and the hesitant fuzzy linguistic TOPSIS method are provided to show the advantages of the proposed method.

The highlights of this paper are summarized as follows:

(1) We define the DHLTS and the DHHFLTS, both of them can be used to describe the linguistic information more accurately.
(2) The DHHFL-MULTIMOORA method with double hierarchy hesitant fuzzy linguistic information, developed in this paper, can comprehensively consider three aspects' information, which ensures the decision making result much more convincing.
(3) This paper mainly solves a practical MCDM problem, which is to select the optimal city in China by evaluating the implementation status of haze controlling measures.

The rest of this paper are organized as follows: We review some concepts and operational laws of HFLTSs in Section 2. In Section 3, we propose the concepts of DHLTS and DHHFLTS, the basic components of which can be denoted as double hierarchy linguistic terms (DHLTs) and double hierarchy hesitant fuzzy linguistic elements (DHHFLEs), respectively. Then two equivalent transformation functions between the DHLTs (DHHFLEs) and the evaluations in [0,1] (HFE) are established. Furthermore, some basic operational laws and properties of DHHFLEs are developed in this section. In Section 4, we first propose a MCDM model with double hierarchy hesitant fuzzy linguistic information, and then develop a novel DHHFL-MULTIMOORA method. In Section 5, we apply the DHHFL-MULTIMOORA method to deal with a practical case about selecting the best city in China by evaluating the implementation status of haze controlling measures. Moreover, we make some comparisons between the DHHFL-MULTIMOORA method and the hesitant fuzzy linguistic TOPSIS method. Finally, we finish this paper with some concluding remarks and future research directions in Section 6.

2. Hesitant fuzzy linguistic terms set: concept and operational laws

In 2010, Torra [1] proposed the concept of HFS on X as a function that when applied to X returns a subset of [0,1]. To be easily understood, Xia and Xu [34] expressed the HFS by a mathematical symbol $A = \{< x, h_A(x) > | x \in X\}$ where $h_A(x)$ is a set of some values in [0,1], denoting the possible membership degrees of the element $x \in X$ to the set A. Additionally, $h = h_A(x)$ can be called a hesitant fuzzy element (HFE) and Θ being the set of all HFEs.

In 2012, Rodríguez et al. [6] defined the concept of HFLTS as an ordered finite subset of the consecutive linguistic terms of a given LTS. Soon afterwards, Liao et al. [13] extended and formalized it mathematically as follows:

Definition 2.1 [13]**.** Let $x_i \in X$ $(i = 1, 2, \ldots, N)$ be fixed and $S = \{s_t | t = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ be a LTS. A HFLTS on X, H_S, is in mathematical form of $H_S = \{< x_i, h_S(x_i) > | x_i \in X\}$, where $h_S(x_i)$ is a set of some values in S and can be expressed as:

$$h_S(x_i) = \left\{s_{\phi_l}(x_i) | s_{\phi_l}(x_i) \in S; l = 1, \ldots, L; \phi_l \in \{-\tau, \ldots, -1, 0, 1, \ldots, \tau\}\right\}$$

with L being the number of linguistic terms in $h_S(x_i)$ and $s_{\phi_l}(x_i)$ $(l = 1, \ldots, L)$ in each $h_S(x_i)$ being the continuous terms in S. $h_S(x_i)$ denotes the possible degree of the linguistic variable x_i to S. For convenience, $h_S(x_i)$ is called a hesitant fuzzy linguistic element (HFLE) and Φ being the set of all HFLEs.

Remark 2.1. Note that, in Definition 2.2, the linguistic terms are chosen in discrete form from S and the subscripts of $s_{\phi_l}(x_i)$, ϕ_l, belong to $\{-\tau, \ldots, -1, 0, 1, \ldots, \tau\}$. In order not to lose much information, there are two well known approaches to extend it to continuous form by using an interval to represent the lateral displacement between two adjacent labels, they are the 2-tuple linguistic model [41] and the linguistic alphabet [42]. In this way, we consider from now on the extension $\phi_l \in [-\tau, \tau]$, which is much general and flexible [42].

Besides, to make the operations of HFLTSs more reasonable, Gou and Xu [7] developed two equivalent transformation functions between the considered interval and the unit interval. Below we improve the definition between the transformation functions between the HFLE and the HFE.

Definition 2.2. Let $S = \{s_t | t = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ be a LTS, $h_S = \{s_{\phi_l} | s_{\phi_l} \in S; l = 1, \ldots, L; \phi_l \in [-\tau, \tau]\}$ be a HFLE with L being the number of linguistic terms in h_S, and $h_\sigma = \{\sigma_l | \sigma_l \in [0, 1]; l = 1, \ldots, L\}$ be a HFE. Then the membership degree σ_l and the subscript ϕ_l of the linguistic term s_{ϕ_l} that expresses the equivalent information to the membership degree σ_l can be transformed to each other by the following functions g and g^{-1}, respectively:

$$g : [-\tau, \tau] \to [0, 1], \quad g(\phi_l) = \frac{\phi_l + \tau}{2\tau} = \sigma_l \tag{1}$$

$$g^{-1} : [0, 1] \to [-\tau, \tau], \quad g^{-1}(\sigma_l) = (2\sigma_l - 1)\tau = \phi_l \tag{2}$$

Based on Definition 2.2, we can introduce the transformation functions between the HFLE h_S and the HFE h_σ.