Applied Soft Computing 11 (2011) 551–560

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

A genetic algorithm for optimization problems with fuzzy relation constraints using max-product composition

Reza Hassanzadeh [a], Esmaile Khorram [b], Iraj Mahdavi [a,*], Nezam Mahdavi-Amiri [c]

[a] Mazandaran University of Science and Technology, Babol, Iran
[b] Faculty of Mathematics and Computer Science, Amirkabir University of Technology, Tehran, Iran
[c] Faculty of Mathematical Sciences, Sharif University of Technology, Tehran, Iran

ARTICLE INFO

Article history:
Received 18 December 2008
Received in revised form
15 November 2009
Accepted 6 December 2009
Available online 16 December 2009

Keywords:
Fuzzy relation equations
Genetic algorithms
Nonlinear optimization
Max-product composition

ABSTRACT

We consider nonlinear optimization problems constrained by a system of fuzzy relation equations. The solution set of the fuzzy relation equations being nonconvex, in general, conventional nonlinear programming methods are not practical. Here, we propose a genetic algorithm with max-product composition to obtain a near optimal solution for convex or nonconvex solution set. Test problems are constructed to evaluate the performance of the proposed algorithm showing alternative solutions obtained by our proposed model.

1. Introduction

Consider the following fuzzy relation equations,

$$xoA = b, \tag{1}$$

where $A=(a_{ij})_{m\times n}$, $0\le a_{ij}\le 1$, is a fuzzy matrix, $b=(b_1,b_2, \ldots, b_n)$, $0\le b_j\le 1$, is an n-dimensional vector and "o" stands for the max-product composition [1], that is,

$$\max_{i=1,\ldots,m} (x_i a_{ij}) = b_j, \quad j = 1, 2, \ldots, n. \tag{2}$$

Given the fuzzy relation matrix A and output vector b, the resolution problem is to determine all input vectors $x=(x_1, \ldots, x_m)$, $0\le x_i\le 1$, satisfying (1). A nonempty solution set of the fuzzy relation equations is generally a nonconvex set determined in terms of the maximum solution and the finite number of minimal solutions [1,2,4–6].

The theory of fuzzy relational equations (FRE) forms a generalization of Boolean relation equations [14]. In [15], Sanchez investigated the notion of fuzzy relation equations based upon the max–min composition. He considered some theoretical methods and conditions to resolve the fuzzy relations. He also presented some results for the determination and existence of solutions of certain basic fuzzy relation equations. The set of solutions of (1) is not usually a singleton. However, he showed that, when the set of solutions is nonempty, it is a nonconvex set, in general, and it can be completely determined by a unique maximum solution and a finite number of minimal solutions. In [16], Sanchez initiated a development of the theory and applications of FRE treated as a formalized model for imprecise notions.

Fang and Li [2] converted an optimization problem with a single linear objective function subject to the fuzzy relation equations based on the max–min composition to a 0–1 integer programming problem and solved it by a branch and bound method. Wu et al. [17] improved Fang and Li's method by providing an upper bound for the branch and bound procedure. Lee and Guu [34] proposed a fuzzy relational optimization model for the streaming media provider seeking a minimum cost while fulfilling the requirements assumed by a three-tier framework.

The max–min composition is normally applied when a system involves conservative solutions in the sense that the goodness of one value cannot compensate the badness of another value [13]. Other compositions can also be used depending on the applications. Yager [18] gives some guidelines for selecting a proper composition.

The fundamental result for the fuzzy relation equations with max-product composition having the conservative property goes

* Corresponding author.
E-mail address: irajarash@rediffmail.com (I. Mahdavi).

1568-4946/$ – see front matter
doi:10.1016/j.asoc.2009.12.014

back to Pedrycz [19]. Another study in this regard can be found in Brouke and Fisher [20]. They extended the study of an inverse solution of a system of fuzzy relation equations with max-product composition. They provided theoretical results for determining the complete set of solutions as well as the conditions for the existence of solutions. Their results showed that such complete set of solutions can be characterized by one maximum solution and a number of minimal solutions. Furthermore, the monograph by Nola et al. [21] contains a thorough discussion of this class of equations. Markovskii [22] showed that solving max-product FRE is closely related to the covering problem, and hence is NP-hard. Chen and Wang [23] designed an algorithm for obtaining the logical representation of all minimal solutions. They showed that a polynomial-time algorithm to find all minimal solutions of an FRE with max–min composition may not exist.

Peeva and Kyosev [26] developed an exact method and a universal algorithm for solving max-product fuzzy linear system of equations and max-product fuzzy relational equations. Markovskii [27] described methods for reducing the dimension of the covering problem and methods for solving fuzzy relation equations with max-product composition.

Linear optimization problems with max-product approach was investigated by Loetamonphong et al. [24]. They defined two subproblems by separating the negative and nonnegative coefficients in the objective function, and then obtained the optimal solution by combining the two subproblems. The subproblem with a negative coefficient is easily optimized by the maximum solution of the set of solutions. The other subproblem was converted to a 0–1 integer programming problem and solved by the branch and bound method. Guu and Wu [25] gave a necessary condition for an optimal solution in terms of the maximum solution derived from the fuzzy relation equations. Guu and Wu [28] studied the optimization problem subject to fuzzy relation equations with max-product composition.

Yang and Cao [29] studied the fuzzy relation geometric programming problems with monomial objective function and fuzzy relation equations as constraints using max–min composition. Guo and Xia [30] presented a new approach for solving optimization problems with one linear objective function and finitely many fuzzy relation inequality constraints.

Tao et al. [31] developed methods for solving the global optimization problem of max–min systems and established the criteria for the existence and uniqueness of global optimal solutions.

Lu and Fang [11] proposed a genetic algorithm to solve a nonlinear single objective problem with fuzzy relation equations as constraints considering the max–min composition.

Here, we consider minimizing a nonlinear objective function constrained by max-product fuzzy relation equations. The set of feasible solutions being nonconvex and the problem having a special structure, we propose to apply a genetic algorithm for finding a solution. The nonlinear programming model with fuzzy relation constraints is formally defined to be:

$$\min f(x), \quad \text{s.t. } xoA = b. \tag{3}$$

In Section 2, we describe our genetic algorithm, in detail. We devote Section 3 to the effective construction of test problems and numerical experimentation. Finally, we conclude in Section 4.

2. The proposed genetic algorithm

Genetic algorithms (GAs) are built upon the mechanism of natural evolution of genetics. GAs emulate the biological evolutionary theory to solve optimization problems. In general, GAs start with a randomly generated population and progress to improve solutions by using genetic operators such as crossover and mutation. In each iteration (a generation), based on the performance (fitness) and some selection criteria, the relatively good solutions are retained and the relatively bad solutions are replaced by some newly generated offsprings. An evaluation criterion (objective) usually guides the selection.

Our proposed GA is designed specifically for solving nonlinear optimization problems with fuzzy relation constraints as specified by (3).

2.1. Representation

In our algurithm, since the solutions of fuzzy relation equations are comprised of nonnegative real numbers not bigger than one then we use the floating point [12] representation in which each gene or variable x_i in an individual $x=(x_1, x_2, \ldots, x_m)$ is a real number in the interval [0, 1].

2.2. Initialization

In general, a GA initializes the population randomly. This works well when dealing with unconstrained optimization problems. However, for a constrained optimization problem, randomly generated solutions may not be feasible. Since GA intends to keep the solutions (chromosomes) feasible, we present an initialization module to initialize a population by randomly generating the individuals inside the feasible domain.

Since some elements will never play a role in determining a solution of the fuzzy relation equations, then we can modify the fuzzy relation matrix by identifying these elements and setting their values to 0 hoping to accelerate the procedure for finding a new solution. To make it clear, we define the “equivalence operation”.

Definition 1. If nullifying (setting to zero) some elements of a given fuzzy relation matrix A has no effect on the solutions of fuzzy relation Eq. (1), then nullifying is called an “equivalence operation”.

The following lemma given in [11] can be useful.

Lemma 1. For $j_1, j_2 \in \{1, 2, \ldots, n\}$, if $b_{j_1}/a_{ij_1} > b_{j_2}/a_{ij_2}$, $a_{ij_1} \geq b_{j_1}$, and $a_{ij_2} \geq b_{j_2}$, for some i, then an equivalence operation can be performed by “setting a_{ij_1} to zero”.

We now give an example to illustrate an equivalence operation.

Example 0. Consider the matrix A and the vector b below:

$$A = \begin{pmatrix} 0.3765 & 0.6539 & 0.6423 & 0.5858 \\ 0.8595 & 0.6044 & 0.5603 & 0.5429 \\ 0.7939 & 0.2591 & 0.3769 & 0.4836 \\ 0.6095 & 0.0260 & 0.1207 & 0.8866 \\ 0.3318 & 0.9870 & 0.4491 & 0.0816 \\ 0.6240 & 0.2077 & 0.1377 & 0.3626 \end{pmatrix},$$

$$b = \begin{pmatrix} 0.6254 & 0.6198 & 0.6010 & 0.8521 \end{pmatrix}.$$

Since $b_2/a_{12} > b_3/a_{13}$, $a_{12} > b_2$, and $a_{13} > b_3$, then the operation “setting a_{12} to 0” is an equivalence operation.

Based on this idea, the initialization module originates a population consisting of a given number of randomly generated feasible solutions. An algorithm for initializing a population is described as having the following steps.