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Notes on shape based tools for treating the objects ellipticity issues

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ABSTRACT

In this paper we put under the same umbrella several well known results and establish several new results related to the shape based tools which treat the objects ellipticity issues. We start with a derivation of an explicit and closed formula for the computation of the ellipticity measures, from an infinite family, introduced recently. The new formula enables a fast computation of such measures, since it does not require any optimizing procedure for the computation, as it was the case before. In addition, the established formula enables an easy theoretical manipulation. As a result, we have discovered new shape features, as they are: (i) *The average shape ellipticity measure*, which might be interpreted as an average value of the estimated similarities between the shape considered and all the ellipses whose axes length ratio belongs to a certain, predefined, interval; (ii) *The maximal shape ellipticity measure*, which might be understood as the maximal possible similarity estimate between a given shape and an ellipses.

Some of the other results obtained relate strongly to the well-known measures and methods broadly used in shape based object analysis tasks.

1. Introduction

The shape is an object property which has a big discriminative capacity since it allows many numerical characterizations. One of common approaches is to observe certain shape descriptors/properties (e.g. convexity, elongation, compactness, sigmodaility, etc.) cognizable and distinct for a certain application, and then develop methods for their numerical evaluation. Such evaluation methods herein are called *shape measures*. There are many shape measures developed so far. Just to mention some of them: convexity [23], circularity[1] [17,22,33], linearity [9,26], tortuosity [10], but there are many more. As it can be seen, there are shape descriptors with multiple measures developed for their numerical evaluation. This is as expected since there is no shape measure performing well in all applications. The shape ellipticity, which is the main subject of this paper, also has multiple measures defined for its computation. Notice that by the shape ellipticity we mean the similarity of a given shape to the planar region bounded by an ellipse. It is worth pointing out that the object/shape ellipticity is a recurrent topic in research, due back to 1910 (see [27]) till the most recent days [16], and many more, e.g. [24,32], just to mention two of them. A lot of work has been dedicated to solve the appearing, ellipticity associated, problems. Here we mention [13,14,19,21], from the areas of the astronomy, astrophysics, nano-particles analysis, and traffic analysis.

Apart from the shape measures mentioned, there are generic shape measures which are targeted to satisfy some of desirable properties (e.g. the invariance with respect to a class of transformations), rather than to evaluate some of shape properties. Among them are: Fourier invariants [29], different kind of moment invariants [5,11,30,31], integral invariants [12], shape-illumination invariants [2], and so on. The power of generic shape invariants comes from the fact that the number of such invariants is not upper bounded. A drawback is that their behavior is not well explained and cannot be predicted.

There another approaches, as well, to the shape analysis problems. Here in we mention statistics based ones (the topic usually named the *statistical shape analysis*. An idea [4] was to learn the space of typical shapes, from examples of a class of shapes (i.e. training data). This can then be used to analyze new shapes, e.g. measure the probability the new shape is a member of the shape class, or transform the new shape by projecting it into the learnt shape space. A statistical appearance model, that uses a probabilistic correspondence (rather than one-to-one correspondences,

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[1] The circularity property is often called *compactness*, since the circular disc is understood as the most compact shape.

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for example) has been considered in [15]. A learning statistical shape model from image data, has been proposed in [8]. The model uses Kendal shape space, represents shapes as point set equivalence classes, and treats each shape point set as a random variable. A method for constructing statistical representations of ensembles of similar shapes is described in [3]. The method is based on an optimal distribution of a large set of surface point correspondences (the method uses surface point samples rather than any specific surface parameterization). Current (a generalized distribution, coming from geometric measure theory) based method for computing an optimal deformation between surfaces embedded in 3D space is given in [28]. The method leads to a diffeomorphic matching algorithm, with an immediate application in statistical inference of shapes, via momentum representation of flow.

As it could be expected, the measures which do relate to certain shape property have a well understood and predictable behavior, but their number is limited. This further causes a limited discriminative power of such measures. An attempt to balance between these two issues has been made in [1], where a tuning parameter $\rho \in (0, 1]$ was involved to design an infinite family of ellipticity measures $\mathcal{E}_\rho(S)$. All measures, from the family $\mathcal{E}_\rho(S)$, with $0 < \rho \leq 1$, are invariant with respect to translation, rotation, and scaling transformations and range over the interval (0, 1]. For a fixed $\rho \in (0, 1]$, the equality $\mathcal{E}_\rho(S) = 1$ is true if and only if the shape S is the ellipse whose the axes length ratio is ρ. Thus, the ellipticity measures $\mathcal{E}_\rho(S)$ distinguish between ellipses whose axes length ratios are different – i.e. such ellipses are considered to be different in shape, as well.

The results of [1] are actually the starting point for the work of this paper. We start with the derivation of an explicit and closed formula for $\mathcal{E}_\rho(S)$.[2] This enables a fast computation and more detailed theoretical observation of the properties of the measures from the family. Based on these observations we discover new shape measures/features, not discussed before in the literature:

- *Average ellipticity*, $\mathcal{E}_{avg}(S)$, is defined as the average score of the measured ellipticities $\mathcal{E}_\rho(S)$, while ρ varies through an interval $(a, b]$, with $0 < a < b \leq 1$.
- *Maximum ellipticity*, $\mathcal{E}_{\max}(S)$, is defined as the maximal value among all the ellipticity measures $\mathcal{E}_\rho(S)$, with $0 < \rho \leq 1$. Formally: $\mathcal{E}_{\max}(S) = \max\limits_{\rho \in (0,1]} \mathcal{E}_\rho(S)$.[3]

Some of the obtained results strongly relate to the well-known results that are already in common use in image processing and computer vision tasks. This will be discussed in more details later on. Several illustrative examples are provided, in order to support a better understanding of the theoretical observations made in this paper.

The paper is organized as it follows: Section 2 gives the basic definitions and denotations. Explicit formulas for the computation of the $\mathcal{E}_\rho(S)$, $\mathcal{E}_{avg}(S)$, and $\mathcal{E}_{\max}(S)$, measures and related comments are Section 3. Concluding remarks are in Section 4.

2. Definitions and denotation

We list shortly the basic terms and denotations, used in the rest of the paper.

- $E(a, b)$ denotes an isothetic ellipse whose axis lengths are a and b, and whose centroid coincides with the origin. Formally,

$$E(a,b) = \left\{(x,y) \mid \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1\right\}. \tag{1}$$

Just as a short reminder, the area of $E(a, b)$ is $\pi \cdot a \cdot b$.

Remark. Instead of the term 'ellipse', in mathematics assumed to be a line (not a region, as in (1)), more correct would be to use the term 'elliptical disc', for the region $E(a, b)$ in (1). However, we will proceed to use the term 'ellipse' because it has been commonly used in literature related to the shape analysis. Similarly, we will use the term 'circle' for planar region bounded by the circular line, instead of, mathematically more correct, term 'circular disc'.

- Since the shape does not change under scaling transformations, without loss of generality, we will assume that all appearing shapes have the area equal to 1.
- $E(\rho)$ will denote an isothetic ellipse, having the unit area and the axes length ratio equal to ρ, and placed such that the centroid of $E(\rho)$ coincides with the origin - i.e.

$$E(\rho) = \left\{(x,y) \mid \frac{x^2}{\left(\sqrt{\frac{\rho}{\pi}}\right)^2} + \frac{y^2}{\left(\frac{1}{\sqrt{\pi \cdot \rho}}\right)^2} \leq 1\right\}$$
$$= \left\{(x,y) \mid \frac{x^2}{\rho} + \rho \cdot y^2 \leq \frac{1}{\pi}\right\}. \tag{2}$$

In other words $E(\rho) = E(a,b)$, with $a = \sqrt{\rho/\pi}$ and $b = 1/\sqrt{\pi \cdot \rho}$.

- Two shapes are said to be equal if their set differences have the area equal to zero. This is not a restriction in practical applications – e.g. a closed region $\{(x,y) \mid x^2 + y^2 \leq 1\}$ and the open one $\{(x,y) \mid x^2 + y^2 < 1\}$ are of the same shape.
- $S(\omega)$ will denote the shape S rotated around its centroid for an angle ω. Notice that the shape centroid (x_c, y_c), as usually, is defined as

$$\left(x_c, y_c\right) = \left(\frac{\iint_S x \, dx \, dy}{\iint_S dx \, dy}, \frac{\iint_S y \, dx \, dy}{\iint_S dx \, dy}\right). \tag{3}$$

- The first two Hu moment invariants [11] will be used intensively in our derivations. They will be denoted by $\mathcal{H}_1(S)$ and $\mathcal{H}_2(S)$ respectively, and are defined as:

$$\mathcal{H}_1(S) = m_{2,0}(S) + m_{0,2}(S) \quad \text{and}$$
$$\mathcal{H}_2(S) = (m_{2,0}(S) - m_{0,2}(S))^2 + 4m_{1,1}(S)^2. \tag{4}$$

The quantities $m_{p,q}(S)$ are so called normalized moments and are defined as

$$m_{p,q}(S) = \iint_S (x - x_c)^p (y - y_c)^q \, dx \, dy, \tag{5}$$

for the shapes having the area equal to 1.

3. Family of ellipticity measures, maximal, and average ellipticity

A family of ellipticity measures $\mathcal{E}_\rho(S)$, dependent on a parameter $\rho \in (0, 1]$, has been introduced recently in [1]. The quantity $\mathcal{E}_\rho(S)$ evaluates the similarity between the considered shape S and the ellipse $E(\rho)$ (whose axes length ratio is ρ - see (2)), and is invariant w.r.t. translation, rotation and scaling transformations. Also, $\mathcal{E}_\rho(S) = 1$ if and only if $S = E(\rho)$.

The ellipticity measures $\mathcal{E}_\rho(S)$ are shown to be very efficient in a galaxy classification task [1]. The elliptical and spiral galaxies listed in the *Nearby Galaxy Catalog* (**NGC**) [7], were used as the data set (see Fig. 1 for some examples). This classification problem is a difficult task. Previous the best accuracy was 95.1% [18]. The 100% classification rate was achieved by employing a number of $\mathcal{E}_\rho(S)$ measures. A simple k-NN classifier was used. In addition, to reduce limits (in the classification efficiency), caused by a choice of the threshold method selected, two shapes were associated to

[2] The formula for the computation of $\mathcal{E}_\rho(S)$, derived in [1], involves an optimizing procedure and is suitable for the numerical computation only.

[3] It may be interesting to point out that $\mathcal{E}_{\max}(S)$ is invariant with respect to affine transformations, while the measures from the family $\mathcal{E}_\rho(S)$ are not.