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Bayesian system reliability assessment under the vague environment

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ABSTRACT

Classical reliability assessment is based largely on precise information. In practice, however, some information about an underlying system might be imprecise and represented in the form of vague quantities. Thus, it is necessary to generalize the classical methods to vague environments for studying and analyzing the systems of interests. On the other hand, Bayesian approaches have shown to be useful when there is some prior information about the underlying model. In this paper, Bayesian system reliability assessment is investigated in vague environments. To employ the Bayesian approach, model parameters are assumed to be vague random variables with vague prior distributions. This approach will be used to create the vague Bayes estimate of system reliability by introducing and applying a theorem called “Resolution Identity” for vague sets. We also investigate a computational procedure to evaluate the vague Bayes estimate of system reliability. For this purpose, the original problem is transformed into a nonlinear programming problem which is then divided up into eight subproblems to simplify computations. Finally, the results obtained for the subproblems can be used to determine the membership functions of the vague Bayes estimate of system reliability. Two practical examples are provided to clarify the proposed approach.

1. Introduction

Our understanding of most physical processes is based largely on vague concepts and imprecise human reasoning. This imprecision is, nonetheless, a form of information that can be quite useful to humans in decision making, control processes, prediction, etc. Imprecision can be used to describe certain kinds of uncertainty associated with linguistic information or intuitive information. Examples of imprecise information are such statements as the product quality is “acceptable”, the lifetime of a lamp is “approximately 2000 (h)”, or the necessary dose of a certain chemical in a drug must be “about 20%”.

Over the past decades, different approaches and theories have been proposed for treating imprecision and uncertainty, among which the fuzzy set theory is a key one. Meanwhile, intuitionistic fuzzy set theory [1] and vague set theory [13] have also been proposed as extensions of the classical fuzzy set theory to investigate the uncertainty in a more realistic manner. Several researchers have concentrated on applying these theories to reliability theory, adopting both classical and Bayesian approaches.

In the present work, the Bayes approach to system reliability estimation is incorporated into the vague set theory to deduce the so-called vague Bayes estimator. First, in Section 2, we briefly review the main attempts in fuzzy reliability and vague reliability. In Section 3, we introduce the concept of vague random variable and investigate the vague Bayes estimator (using the Wu’s approach [36]) that is constructed from the conventional Bayes estimators by applying a “Resolution Identity” for vague sets. In Section 4, first we recall the concept of Mellin transformation and review the Bayesian approach to the problem of system reliability. Then, we discuss the vague Bayes estimator of system reliability for series system and parallel systems. In Section 5, the computational procedures for dealing with the proposed approach are provided. Two practical examples related to system reliability are presented in Section 6. Summary and conclusions are provided in Section 7.

2. A review of fuzzy reliability and vague reliability studies

In this section, we review the main studies on fuzzy reliability and vague reliability (also called intuitionistic fuzzy reliability) in an attempt to display the motivation for this paper.

To the best of our knowledge, application of fuzzy probability to fault-tree analysis was first studied by Tanaka et al. [27] and by Furuta and Shiraishi [12]. Singer [24] presented a fuzzy set approach to fault-tree analysis in which the relative frequencies of the basic events are considered as fuzzy numbers. Cai and Wen [3] introduced the fuzzy success state and fuzzy failure state in which a transition between two fuzzy states was regarded as a fuzzy event.

Cai et al. [4] pointed out that there are two fundamental assumptions in the conventional reliability theory:

(1) Binary state assumptions: the system is precisely defined as either functioning or failing.

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(2) Probability assumptions: the system behavior is fully characterized in the context of probability measures.

However, due to inaccuracies or uncertainties in data, it is very difficult in many systems to estimate precise probability values. This caused Cai et al. [5,7] to present the following two assumptions and to study certain problems in fuzzy reliability concerning such assumptions:

(1) Fuzzy-state assumption: The meaning of the system failure cannot be precisely defined in a reasonable way. At any time the system may be in one of the following two states: fuzzy success state or fuzzy failure state.
(2) Possibility assumption: The system behavior can be fully characterized in the context of possibility measures.

Also, they classified theories of fuzzy reliability along the following lines [6]:

(1) PROFUST reliability theory, based on the probability assumption and fuzzy-state assumption.
(2) POSBIST reliability theory, based on the possibility assumption and binary state assumption.
(3) POSFUST reliability theory, based on the possibility assumption and fuzzy-state assumption.

Cheng and Mon [11] and Mon and Cheng [17] proposed a methodology for fuzzy system reliability using the α-level sets of fuzzy numbers. However, their method obtained only the α-level set (the interval of confidence) for a prescribed value $\alpha \in [0, 1]$. In fact, they did not mention how to come up with the membership degree for any given system reliability.

Utkin [28–30] discussed the fuzzy system reliability based on the binary state assumption and possibility assumption. He considered the fuzzy availability and unavailability and the fuzzy operative availability and unavailability to be described below:

(i) A fuzzy availability is a possibility that the system is in the functioning state at time t. A fuzzy unavailability is a possibility that the system is in repair at time t. The fuzzy availability and unavailability are characterized by possibility distribution functions.
(ii) A fuzzy operative availability is a possibility that the system is in the functioning state at time t and continues to function without failures during a fuzzy time interval $[t, t+\tilde{s}]$, where $\tilde{s}$ is a fuzzy number. A fuzzy operative unavailability is a possibility that the system is in repair at time t and continues in this state during a fuzzy time interval $[t, t+\tilde{s}]$. Fuzzy operative availability and unavailability are characterized by possibility distribution functions.

Chen [9] presented another method for analyzing the fuzzy system reliability based on fuzzy arithmetic operations. His method is based on the PROFUST reliability theory, where the reliability of each system component is represented by a triangular fuzzy number. Viertl and Gurker [34] considered the reliability estimation with fuzzy lifetime data (see also Viertl [31]). In addition, Viertl [32] generalized some classical and Bayesian procedures to reliability estimation based on fuzzy data. Wu [35] proposed to use the fuzzy probability for studying a system's reliability. He discussed a system reliability in which the probability of each component in the system was assumed to be a fuzzy number. Also, he introduced the fuzzy conditional probability to derive the pivotal decomposition based on the fuzzy reliability and improved the fuzzy reliability of the system by improving the fuzzy probability of each component under fuzziness. The drawback of the above mentioned models is their lack of a supporting theory of fuzzy random variables. On the other hand, the parameters in their approaches are not assumed to be fuzzy parameters, while it will be more natural to consider fuzzy estimators when the parameters are fuzzy.

From a different perspective, using the concept of fuzzy random variable, Wu proposed a Bayesian approach to system reliability assessment under the fuzzy environment [36–38]. According to his approach, the fuzzy parameters are assumed to be fuzzy random variables with prior fuzzy distributions. He investigated the fuzzy Bayes point estimators of system reliability based on exponential distribution for series system, parallel systems, and k-out-of-n systems. In addition, Huang et al. [14] investigated the Bayes approach for the analysis of fuzzy lifetime data. In fact, they proposed a method to determine the membership function of the estimates of the parameters and the reliability function of the multi-parameter life time distributions (for Bayes approaches in fuzzy environments, see for example [26,33]).

Adopting a different approach, Sasaki and Gen [19] presented a method for solving fuzzy multiple objective optimal system design problems. This method allows for a flexible optimal system reliability design by applying fuzzy goals and fuzzy constraints.

Recently, Sharma et al. [22] proposed a structured framework for predicting uncertain behavior of industrial systems using a fuzzy methodology. In their work, various parameters of managerial importance such as repair time, failure rate, mean time between failures, availability and expected number of failures are computed to quantify the uncertain behavior of system.

Once the vague (intuitionistic fuzzy) set theory have been introduced, a number of authors investigated the reliability analysis in vague environments. Chen [10] presented a method to analyze system reliability using the vague set theory, where the reliabilities of system components are represented by vague sets. Kumar et al. [15] developed a method for analyzing system reliability by using interval-valued vague sets, and applied it for the reliability analysis of a Marine Power Plant. They also extended the above method using T_w (the drastic t-norm) based arithmetic operations [16]. In the proposed method, an interval-valued vague fault-tree is developed for the system in which the fuzzy reliability of each component of the system is represented by a LR type interval-valued triangular vague set. Then, using this interval-valued vague fault-tree, an algorithm is developed to analyze the fuzzy system reliability. Also, Shu et al. [23] studied the fault-tree analysis using vague sets. They proposed an algorithm to calculate the fault interval of system components by integrating both expert knowledge and experience in terms of providing the possibilities of failure of bottom events. They applied their method to the failure analysis problem of printed circuit board assembly (PCBA). A more or less similar approach has been considered by Chang et al. [8] to develop vague fault-tree decision support systems (VFTDSS) to generate fault-tree and fault-tree nodes, and to compute the vague fault-tree interval, traditional reliability, and vague reliability interval.

As mentioned earlier, a Bayesian approach to system reliability is developed in the present work based on the vague set theory.

3. Vague Bayes point estimator

Definition 1. Let U be a universal set. A vague set $\tilde{A}$ of U is characterized by a truth membership function $t_{\tilde{A}} : U \to [0, 1]$, and a false membership function $f_{\tilde{A}} : U \to [0, 1]$, where $t_{\tilde{A}}(u) + f_{\tilde{A}}(u) \le 1$, $\forall u \in U$.

In the above definition, $t_{\tilde{A}}(u)$ is considered as the lower bound for the degree of membership of u in $\tilde{A}$ (based on evidences), and $f_{\tilde{A}}(u)$ is the lower bound for the negation of membership of u in $\tilde{A}$. Therefore, the degree of membership of u in the vague set $\tilde{A}$ is characterized by the interval $[t_{\tilde{A}}(u), 1 - f_{\tilde{A}}(u)]$. A typical illustration