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Convex hull indexed Gaussian mixture model (CH-GMM) for 3D point set registration

Jingfan Fan, Jian Yang*, Danni Ai, Likun Xia, Yitian Zhao, Xing Gao, Yongtian Wang

Beijing Engineering Research Center of Mixed Reality and Advanced Display, School of Optics and Electronics, Beijing Institute of Technology, Beijing 100081, China

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ABSTRACT

To solve the problem of rigid/non-rigid 3D point set registration, a novel convex hull indexed Gaussian mixture model (CH-GMM) is proposed in this paper. The model works by computing a weighted Gaussian mixture model (GMM) response over the convex hull of each point set. Three conditions, proximity, area conservation and projection consistency, are incorporated into the model so as to improve its performance. Given that the convex hull is the tightest convex set of a point set, the combination of Gaussian mixture and convex hull can effectively preserve the topological structure of a point set. Furthermore, computational complexity can be significantly reduced since only the GMM of the convex hull (instead of the whole point set) needs to be calculated. Rigid registration is achieved by seeking the best rigid transformation parameters yielding the most similar CH-GMM responses. Non-rigid deformation is realized by optimizing the coordinates of the control points used by the thin-plate spline model for interpolating the entire point set. Experiments are designed to evaluate a method's robustness to rotational changes between two point sets, positional noise, differences in density and partial overlap. The results demonstrated better robustness and registration accuracy of CH-GMM based method over state-of-the-art methods including iterative closest point, coherent point drift and the GMM method. Besides, the computation of CH-GMM is efficient.

1. Introduction

Pairwise point set registration is an essential component in many computer vision applications, including model retrieval, stereo matching, model mosaic, motion tracking and shape recognition [1]. Pairwise registration aims to estimate the best transformation that aligns two models based on the shapes of their overlapping components. Over the past several decades, numerous 3D registration methods have been proposed. The iterative closest point (ICP) method by Besl et al. [2] is the most popular due to its simplicity and speed. Given point correspondence, transformation is updated by solving a least square problem and given transformation, point correspondence is recovered based on the nearest neighbor relationship. But the one-to-one hard assignment assumption of the method causes the optimization not to be stable and the method can easily be trapped in local minima.

In a different manner, the Gaussian mixture model (GMM) [3–11] based method works by modeling point sets using probability distributions and registration is achieved by seeking the transformation so that the two point sets exhibit similar GMM responses. The GMM-based method has been proven to be effective for point set registration. However, for point sets with similar shape but different densities or noise levels, the GMM responses can differ considerably. Consequently, the GMM-based methods can be trapped in local minima during optimization.

To address the difficulties of the above methods, a novel convex hull indexed Gaussian mixture model (CH-GMM) is proposed in this paper for non-rigid registration of 3D point sets. The CH-GMM model is constructed in the following way: first, local positional noise is smoothed and coplanar points are removed through a principal component analysis (PCA)-based resampling method [12]. Then, the convex hulls of both point sets are robustly extracted by using the quick-hull algorithm [13]. Finally, the CH-GMM response is calculated based on the Gaussian mixture function over the convex hull of a point set, where three conditions, proximity, area conservation and projection angle, are considered to improve performance of the method. As a result, the CH-GMM response will remain consistent for point sets with similar shapes but different densities. Based on the proposed CH-GMM model, rigid registration is achieved by seeking the rigid transformation parameters yielding the most similar CH-GMM

* Corresponding author. Tel./fax: +86 10 68912565-18.
E-mail address: jyang@bit.edu.cn (J. Yang).

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responses for the two point sets, and non-rigid registration is achieved by optimizing the thin-plate spline (TPS) interpolation parameters of the vertices of the convex hull.

The major contributions of the paper are summarized as follows: first, the proposed CH-GMM model can be seen as an improvement of GMM to make the GMM measurement consistent for point sets with similar shapes but different densities. Secondly, the proposed CH-GMM based registration method can effectively prevent the iterative optimization process from falling into local minima, thus improving the stability of the registration method. Finally, since the construction of CH-GMM only involves convex hull, the computational complexity can be significantly reduced compared with state-of-the-art methods.

2. Related works

To improve registration accuracy, numerous studies have preferred using numerous features to construct the corresponding relationship of a point set; these features include the normal vectors of surfaces [14,15], local invariant features [16,17], curvatures [18–20], spin images [21], and textures [22–26]. Feature-based algorithms aim to establish a pairwise relationship (feature extraction) and to distinguish a limited number of invariable features from numerous points with respect to similarity transformation (matching). However, feature extraction and matching procedures are sensitive to imaging noise and resolution. Hence, a series of shape-based registration approaches has been proposed. The underlying shape representation [27,28] is generally considered the most important component in registering arbitrary shapes. Deformable templates [29], Fourier descriptor [30], the snake model [31], and level set representation [32] have been proven to be powerful in estimating several local deformations; however, they require many parameters to represent shape deformation, which is time-consuming. Belongie et al. [33] introduced a shape context-based registration method that established correspondence by incorporating the neighborhood structure of point sets. However, this method is insufficiently robust for a wide variety of real objects. Zheng et al. [34] formulated the registration procedure as an optimization problem to preserve local neighboring structures. For this algorithm, each mesh point of a structure is interpreted as a node in a simple graph, and two neighboring nodes are connected by an edge. The matching of two graphs is then transformed to maximize the number of matched edges. This method utilizes many continuous optimization techniques for registration. However, this procedure may result in many outliers or considerable occlusion; that is, a significant degeneration of local structures may occur, which can lead to algorithm failure. Paragios et al. [35] suggested a signed distance transform for shape alignment that could be extended to any dimension. For this algorithm, signed distance transforms are utilized to constrain the optimization criterion to achieve global and local pixel-wise deformation estimation. Ma et al. [36] adopted L_2 minimizing estimation to approximate the transformation from correspondences for non-rigid point set registration. This method can recover point correspondences iteratively and remove noise and outliers effectively. Chui et al. [37–39] proposed TPS robust point matching to estimate non-rigid transformations between point sets by using spatial mapping and outlier rejection to establish correspondences and to optimize warping parameters. Lian et al. [40,41] suggested that the traditional robust point matching method could be reduced to a concave function by eliminating transformation variables and applying linear transformation. Concave optimization benefits correspondence problems and achieves a globally optimal solution without regularizing transformations. Deng et al. [8] cast the point cloud matching problem as a Schrodinger distance transform (SDT) representation problem. SDT is a measure of square root densities; it ensures the identification of shape distance with geodesic length and enables geodesic length to be expressed as a compact analytic expression on a unit Hilbert sphere. Aiger et al. [42] developed a four-point congruent set approach by extracting all coplanar four-point sets from source point sets that were approximately congruent with a given set of coplanar four points in the target point set under rigid transformation.

As mentioned earlier, most ICP or shape-based algorithms can be classified generally as hard assignment methods, which require the accurate calculation of corresponding pairs between different objects. However, solving the correspondence issue is difficult, particularly when point sets are affected by noise and nonlinear deformations. By contrast, the soft assignment technique has been proposed for fuzzy correspondence optimization; it allows for one-to-many relaxations or uncertain correspondences. Filtering methods are a means to realize soft assignment. Ma et al. [43,44] proposed the use of a particle filter for rigid point set registration. The unscented Kalman filter (UKF) and Gaussian filters were utilized to determine the Gaussian approximation of the posterior for each particle. However, the algorithm requires numerous particles to perform accurate registration, which entails high computational costs for large data sets. Moghari et al. [45] introduced the UKF algorithm to register the rigid transformation of two point sets corrupted with additive Gaussian noise. The transformation parameters were reformulated as state vectors and estimated using UKF, which utilized the unscented transform and the true nonlinear model to approximate the distribution of the state random variable. The algorithm was evaluated based on point sets extracted from a pelvic cadaver through computed tomography (CT) and a scaphoid bone phantom through tracked free-hand ultrasound imaging. Sandhu et al. [46,47] introduced particle filtering and stochastic dynamics for rigid point set registration. In their study, the registration process was constrained by the non-parametric prediction of filtering, and the uncertainty of transformation estimation was solved using dynamic schemes. This method requires no annealing schedule, and thus, it can reduce computational complexity and maintain the detailed information of point sets.

The probability density distribution approach is also a type of soft assignment technique. It assumes that each point of a source model corresponds to a weighted sum of the target points under a constraint of probability density distribution, instead of the closest target point alone; therefore, point set registration can be considered a means of aligning two density distributions by minimizing their discrepancies. Goldberger et al. [48] regarded the Kullback–Leibler divergence between two Gaussian mixtures of registered images, which was utilized to estimate the unscented transform between the images, as a similarity measure. On the basis of kernel correlation theory [3], the GMM proposed by Jian et al. [4,5] demonstrated that point set registration could be reformulated as the static alignment of two Gaussian mixtures, and the best transformation could be obtained by minimizing the discrepancy measure between two corresponding mixtures. From this framework, they used distance to calculate rigid and non-rigid transformations between two point sets. However, GMM is known to be excessively sensitive when data points are few because of its high sensitivity to outliers. To address this problem, Gerogiannis et al. [11] proposed a student's t-mixture model (SMM), which originated from a wide class of elliptically symmetric distributions with the number of degrees of freedom. SMM obtains the corresponding partitioning of image pixels into clusters, and each cluster is represented by a corresponding density component. This model leads to a more robust outcome on outlying pixels. Wassermann et al. [7] presented an anatomical structure registration