Pattern Recognition Letters 100 (2017) 1–5

Contents lists available at ScienceDirect

Pattern Recognition Letters

journal homepage: www.elsevier.com/locate/patrec

Novel phase-based descriptor using bispectrum for texture classification

Lagdali Salwa*, Rziza Mohammed

LRIT CNRST URAC29, Mohammed-V University, Faculty of science, 4, Av Ibn Battouta, BP 1014, Rabat, Morocco

ARTICLE INFO

Article history:
Received 17 March 2017
Available online 18 September 2017

Keywords:
Bispectrum
Phase
Radon projections
Circular statistics
Invariant features
Texture classification

ABSTRACT

In this paper, we propose a novel rotation invariant and noise tolerant descriptor for texture classification. This descriptor is based on circular statistics of Fourier phase. This one is recovered from the third order spectrum namely the bispectrum, known to provide invariance properties and to preserve phase information. The computational complexity of two dimensional image is reduced using Radon transform. At first, the input image is decomposed into a set of 1D radon projections. Then, for each projection the bispectrum is computed and the phase vector is recovered. Features vectors contain circular statistics of each phase vector recovered from bispectrum of each 1D projection. The proposed descriptor is evaluated on three test suites from the database “Outex” and compared with three descriptors also based on the phase. According to the classification experiments, our descriptor achieves highest rates under noise, illumination and rotation changes.

1. Introduction

Texture analysis is an important area of study in image processing which seeks to find an efficient description of textures. Several methods for texture representation have been proposed in the literature. Since repetitiveness is an important characteristic of the texture, it is therefore reasonable to expect a frequency based representation of the texture in terms of magnitude and phase components. As stated in [16], the most valuable information of an image resides in phase and not in magnitude. Recently, the phase information in images plays more important role in various tasks such as texture image retrieval [13], facial recognition [4], biomedical engineering [5,17] and texture analysis [14]. However, these methods describe phase in terms of autocorrelation, which means second order statistics, so they assume Gaussianity of images which is not the case in reality as proved in [10] and lose all the phase information. In contrast, Higher Order Spectra (HOS) do contain such informations due to their important properties which are:

- The ability of capturing and preserving the Fourier phase of 2D processes that are non Gaussian.
- Higher order spectra of a Gaussian filed is zero which allows to suppress Gaussian noise and detect Fourier phase in high signal-to-noise ratio domain.
- Robust against geometrical distortions.

These important properties yield higher order spectra and especially the third order namely the bispectrum, to become an interesting and useful tool in several fields in image such as: pattern recognition [6], image restoration [19,20], texture classification[9] and biomedicine [1] [2]. In this paper, we will retrieve the Fourier phase from bispectrum to propose a new descriptor for texture classification based on circular statistics. Given the high amount of time needed to compute the bispectrum information, the input images are decomposed into a set of 1D projections using the Radon transform. Once image decomposition achieved, the bispectrum information is computed from each 1D image projection obtained. The phase information is therefore recovered from the bispectrum. The circular statistics of the phase are then calculated and used as input for the Support Vector Machine (SVM)[1] classifier. In order to evaluate our procedure, classification tests are performed on textures from the famous database Outex[2], and compared with the prominent methods in texture classification also derived from the phase information as:

- *Local phase quantization (LPQ)* [18] uses the phase information computed locally in a window for every image position.
- *Rotation invariant local phase quantization (RILPQ)* [15] uses local frequency Fourier phase by estimating the local orientations and then compute the directed binary descriptor.

* Corresponding author.
E-mail address: salwalagdali@gmail.com (L. Salwa).

[1] https://www.csie.ntu.edu.tw/~cjlin/libsvm/.
[2] http://www.outex.oulu.fi/.

http://dx.doi.org/10.1016/j.patrec.2017.09.027

- *Rotation invariant local frequency descriptor (RILFD)* [12] uses the phase features of the Local frequency components defined on a circle at each pixel.

The remainder of this paper is organized as follows: The main functions and properties of bispectrum are described in Section 2. Our phase features extraction procedure is detailed in Section 3. Classification tests and results are exposed in Section 4. Finally, Section 5 concludes our work.

2. Bispectrum: definition and properties

The bispectrum of a 1D signal x(t) is the Fourier transform of the triple correlation. With the convolution theorem it can be defined as:

$$B(f_1, f_2) = X(f_1).X(f_2).^{*}X(f_1 + f_2) \tag{1}$$

where X denotes the Fourier transform of the signal x and $^{*}X$ its conjugate at frequency f.

Unlike the power spectra, higher order spectra as in the case of bispectrum satisfy invariance properties [7] such as:

- **Rotation invariance:**
  A rotation does not change the analysed image since the phase values are summed in the product of Fourier coefficients from the equation of bispectrum as proved by Eq. (2).
  Let $B_x(f_1, f_2)$ and $B_{x_\theta}(f_1, f_2)$ be respectively the bispectrum of x and the bispectrum of the rotated version.

$$\begin{aligned} B_{x_\theta}(f_1, f_2) &= X_\theta(f_1).X_\theta(f_2).^{*}X_\theta(-f_1 - f_2) \\ B_{x_\theta}(f_1, f_2) &= X(f_1).e^{-2j\theta f_1}.X(f_2).e^{-2j\theta f_2}.X(f_1+f_2).e^{-2j\theta(-f_1-f_2)} \\ B_{x_\theta}(f_1, f_2) &= X(f_1).X(f_2).^{*}X(-f_1 - f_2) \end{aligned} \tag{2}$$

- **Noise immunity:** The bispectrum of a Gaussian process is zero. In other word Gaussian noise is suppressed in higher order spectra.
- **Phase preservation:** The bispectrum is also known to preserve phase information unlike power spectrum that preserves only amplitude information. Due to the importance of Fourier phase in image processing , and the loss of this information through the conjugate multiplication. We use bispectrum to recover the phase lost in the power spectra.

In this paper we are interested in the phase information retrieved from the bispectrum. The recovery algorithm is detailed in Section 3.

3. The proposed method

Given the fact that the phase contains the essential informations of the image [16] and this information is lost through the spectrum, the phase used in our approach is derived from the bispectrum known to retain it.

A schematic block diagram of our method is depicted in Fig. 1. Firstly, we decompose the image into a set of 1D projections using Radon transform. After, we estimate the bispectrum for each projection. Then, we recover the phase information for each bispectrum. Finally, we compute the circular statistics (M,V,S,K) of each phase. These statistics will feed our features vector and will be used as an input of the SVM classifier.

Precisely, our proposed method involves three steps that we explain in the following subsections.

3.1. Radon transform

The first step in our algorithm is calculating the bispectrum of a given 2D image. Since the bispectrum of a 1D signal is a 2D function, the bispectrum of a 2D signal is a 4D function as shown by Eq. (3) where I is the 2D Fourier transform of an image:

$$B(f_1, f_2; f_3, f_4) = I(f_1, f_2).I(f_3, f_4).^{*}I(f_1 + f_2; f_3 + f_4) \tag{3}$$

If the computational complexity of the Fourier transform of an image with size $(N \times N)$ is $O(N \times N)$, so the computational complexity of the estimation of the 4D bispectrum is $O(N \times N)^3 = O(N^6)$. This requires enormous amount of operations and time.

To reduce the computational complexity, we use the Radon transform which decomposes a 2D image into a set of 1D projections at $\theta_{i=1...M}$ angles. The radon transform $r(d, \theta)$ of an image i(x,y) is defined as:

$$r(d, \theta) = \int\int i(x, y)\delta(r - x\cos\theta - y\sin\theta)dxdy \tag{4}$$

Where $\delta(.)$ is the Dirac function and r is the perpendicular distance from a line to the origin.

For each projection $p_i = r(d, \theta_i)$ of length N we compute the averaged bispectrum as:

$$B_i(f_1, f_2) = P_i(f_1).P_i(f_2).^{*}P_i(f_1 + f_2) \tag{5}$$

where $B_i(f_1, f_2)$ is the bispectrum of the i^{th} projection.

In this case the estimation of the bispectrum needs $O(M)O(N^3)$ operations.

The use of Radon transform in the estimation of the bispectrum do not just reduce the computational complexity but also extend the aforementioned properties of the bispectrum to 2D data as in the case of textures where:

- Due to the invertibility of the Radon transform, there is no loss of the image information.
- The rotation of the image results in a circular shift of the projections along θ. Therefore computing the bispectrum after applying Radon transform produce a rotation invariant.
- The radon transform preserves the directional information and the pixel intensities of the image.

3.2. Phase recovery

The second step of our algorithm consists on recovering the phase information from the bispectrum of each projection. Since $B(f_1, f_2)$ and $P(f)$ are complex values, Eq. (5) can be written otherwise in terms of magnitude (real part) and phase (imaginary part) as:

$$\begin{aligned} &|B(f_1, f_2)|.e^{i\psi(f_1, f_2)} = |P_i(f_1)|.e^{i\phi(f_1)}.|P_i(f_2)|.e^{i\phi(f_2)}. \\ &^{*}|P_i(f_1 + f_2)|.e^{-i\phi(f_1+f_2)} \end{aligned} \tag{6}$$

Hence, we obtain the following equation which contains the object phase where:

$$e^{i\psi(f_1, f_2)} = e^{i[\phi(f_1)+\phi(f_2)-\phi(f_1+f_2)]} \tag{7}$$

Accordingly, Biphase (bispectrum phase) could be defined as the sum of three Fourier phases ϕ by:

$$\psi(f_1, f_2) = \phi(f_1) + \phi(f_2) - \phi(f_1 + f_2) \tag{8}$$

The relation above represents a recursive equation. If the phase of the object at f_1 and f_2 are known, the phase at $(f_1 + f_2)$ can be calculated using:

$$\phi(f_1 + f_2) = \phi(f_1) + \phi(f_2) - \psi(f_1, f_2) \tag{9}$$

As described above we determine the Fourier phase recovered from bispectrum for each projection of the image.