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Image representation and matching with geometric-edge random structure graph☆

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ABSTRACT

Structure graphs are often used in image structural representation by organizing the units of image (such as feature points). However, due to “noise” or non-rigid deformations, the graphs generated from images are usually not stable. To overcome this problem, image matching and recognition can usually be achieved by inexact graph matching means. There has been recent much work on inexact graph matching, but not much on robust graph modeling itself. In this paper we develop a new robust structure graph model for image representation and matching. We believe that a robust structure graph model should adapt to the noise or perturbation of the image units. Here, we explore random graphs instead of traditional graph models and propose a novel random structure graph, called Geometric-Edge random graphs (G-E graphs), for image representation and matching. The main idea of G-E graphs is that the probabilities of edges between node pairs are explored to indicate the uncertainty or variations of edges in the geometric graph generated under some noise or perturbation of the image units. Promising experimental results on both image matching and pattern space embedding show that the proposed G-E graphs are effective and robust to structural variations and significantly outperform traditional graph models.

1. Introduction

Image modeling and representation is very important in image processing and recognition. There are voluminous literatures on methods of image modeling and representation. Generally, these methods can be divided into two categories, i.e., spatial methods and transformation methods. For spatial methods, traditional image models usually include polygon, skeleton, chain code, run length code, pyramid, and high level graphs [1–3,6,9,23]. The advantages of these graph models are twofold: (1) they allow to describe structural relationships between image units, such as regions, feature points and so on. (2) From mathematical aspect, there exist solid problem solving or analyzing schemes. When images are modeled by graphs, image matching and recognition can be converted to graph matching and recognition [2,10,12,13,22,24,25,34,36,37]. Also, when shape images are represented by graphs, shape matching and retrieval can be conducted via shape graph matching [5,14,30]. Many graph models have been proposed to represent images and object shapes. The widely used models include region adjacency graph [23], minimum spanning tree (MST), Delaunary graph [4,24,25], KNN graph [1], geometric graph, shock graph [5,30] and so on. In this paper, we focus on feature points based image representation and matching.

When feature points are extracted from an image based on Harris or SIFT like technique, structure graphs, such as Delaunay graph, KNN graph and geometric graph, can be constructed to describe some structural information of this image. However, due to some non-rigid deformations or imperfect feature point extraction process, the feature points generated from image usually have “noise” or minor variations. Therefore, the structure (edges) of generated graph may not be stable. As shown in Fig. 1, some edges of generated geometric graph are changed (notice the red lines) due to the variations (perturbations) of the point positions. This indicates that there exists some uncertainty and variations in the graph structure. In order to deal with the uncertainty of edge’s existence, one popular way is to use inexact graph matching methods. Recently, many inexact graph matching works have been proposed by using some advanced optimization methods [15,20,26,40]. In addition to optimization methods, probabilistic and evolution techniques have also been used for inexact graph matching problem [1,13,41]. Another way to deal with the uncertainty of the edge’s existences is to encode it by a probability associated to each edge of the graph [39].

Our aim in this paper is to develop a new random graph model for image structure representation and matching. Different from previous work [39], the core idea of the proposed graph

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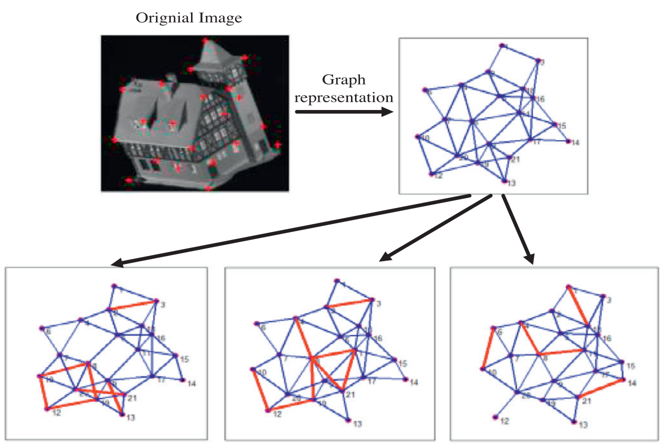

Fig. 1. Top: image feature points extraction (left) and associated geometric graph representation (right). Bottom: the geometric graphs with changeable structures (red edges) due to variation of points positions. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

random model is to describe the uncertainty of edge's existences by using a general probability distribution which is more effective and sufficient than the single probability value. Then, we develop an effective matching process in which the edge compatibilities between edges of two graphs are defined by using a similarity function between edge's distributions. Note that random graphs have been explored for object representation and recognition tasks in previous works [6,31,32,33,36]. Differently, here we focus on structure graphs instead of attributed relation graphs studied in previous works. The proposed random structure graph can be regarded as a natural extension of traditional structure graphs.

The main contributions of this paper are as follows.

(1) Inspired by random geometric graph (RGG) [28] and generalized edge random graph (GERG) [7,19], we developed geometric-edge random graph (G-E graph) model for image representation. In G-E graph, the uncertainty of the edges is introduced by further considering the perturbations of the image feature points. Therefore, it can adapt to the perturbation or noise of the image feature points in nature, and thus can capture the structural variations of image caused by the "noise" or minor deformations effectively.
(2) By introducing adjacency probability matrix, we provide a matrix representation for G-E graph. Thus, we can analyze and match G-E graphs effectively by adopting some algebraic and matrix methods such as spectral graph methods, algebraic graph theory and matrix factorization techniques [20,24,25]. This further provides the practicality of the proposed G-E graph model.
(3) By further incorporating the distance information into the G-E graph, we extend the G-E graph to the weighted form which is noted as W-G-E graph. Some matching techniques are adopted for both G-E and W-G-E graphs. Comparing with G-E graph, W-G-E graph contains more structural information which will be shown more beneficial in image matching.

To verify the effectiveness and robustness of the proposed model, we first pursue image matching. We will show that the proposed model significantly outperforms the traditional graph models in this task. Then, we embed images into low-dimensional pattern space based on the spectral embedding of graphs. We will show that G-E graph can distinguish the image classes more effectively in pattern space. Also, it can preserve the neighborhood relationship in the pattern space for the images under different view angles. Our previous work on G-E graph model has been proposed in our work [22]. In this paper, we give the deeper explanations and justifications for the G-E graph. In particular, we extend G-E graph to a weighted form and propose W-G-E graph model. Also, we conduct a thorough experimental evaluation on both graph matching and spectral embedding to demonstrate the benefits of the proposed model.

Our work is similar to the work of Carreira-Perpiñán and Zemel [11] which propose a Perturbed MST (PMST) graph model for data clustering and manifold learning. The main differences between our G-E graph and PMST are two aspects: (1) Different from PMST, which provides a perturbation model for MST, our G-E graph can be regarded as a perturbation model for geometric graph. More importantly, we discuss this perturbation from the random graph (probabilistic) perspective and provide more theoretical justification. (2) In G-E graph, the edge probability can be computed explicitly. This provides more statistical justification to the proposed G-E graph model.

2. Related work

In this section, we briefly summarize some related graph models [7,22,28].

2.1. Geometric graph

Let $||\cdot||$ be some norm (such as Euclidean norm) on R^d. Given a finite point set $\mathbf{Y}=\{\mathbf{y}_1, \mathbf{y}_2\ldots \mathbf{y}_n\}$ on R^d and a positive parameter r, the geometric graph is determined as follows. Let $V=\{v_1, v_2,\ldots, v_n\}$ be the node set, and $\mathbf{A}(r)$ denotes the adjacency matrix with the element $\mathbf{A}(r; i, j)$ denoting the adjacency relationship between nodes v_i and v_j. Each node v_i in V represents the point $\mathbf{y}_i$ in $\mathbf{Y}$, and an edge exists between nodes v_i and v_j if $||\ \mathbf{y}_i - \mathbf{y}_j\ ||\le r$, i.e.,

$$A(r; i, j) = \begin{cases} 1 \text{ if } ||\mathrm{y}_i - \mathrm{y}_j|| \le r \\ 0, \text{otherwise} \end{cases}$$

In the following, r corresponds to the geometrical parameter of our method. In image matching and recognition, geometric graph as well as Delaunay and kNN graphs can be used to organize image feature points and extract structural information for the image [1,3,24].

2.2. Random geometric graph

The idea of random geometric graph (RGG) has been introduced in the works [7,28]. Here, we give a brief introduction from the perturbation point of view. In geometric graph, the node set $\mathbf{Y}=\{\mathbf{y}_1, \mathbf{y}_2,\ldots,\mathbf{y}_n\}$ contains n points on R^d. Considering the perturbation of point positions due to some "noise" or minor deformations, the perturbed coordinate for point i is denoted as $\mathrm{x}_i = \mathrm{y}_i + \varepsilon_i$, where $\varepsilon_i \in R^d$ is a random d-dimensional noise variable. Let $\mathbf{X}=\{\mathbf{x}_1, \mathbf{x}_2,\ldots, \mathbf{x}_n\}$, then random geometric graph $\mathbf{G}(\mathbf{X}; r)$ is an ensemble of geometric graphs with node point $\mathbf{x}_i$ ($i=1, 2,\ldots,n$) taking different values on R^d under the perturbation noise ε_i. Different from geometric graph, RGG model further tackles the variations of point positions. RGG is a vertex random graph (VRG) [7]. For VRGs, all the randomness inhabits in the structures attached to the graph nodes. Once the random variables have been assigned to the vertices, the edges are determined.

2.3. Generalized edge random graph

Besides VRG, another kind of random graph model is the generalized edge random graph (GERG) [7,19]. Let $V=\{v_1, \mathrm{v}_2,\ldots,\mathrm{v}_n\}$ be the node set, and $\mathbf{P}$: $[1,\ldots,n]\times[1,\ldots,n]\to[0,1]$ be a symmetric