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Multilayer matching of metric structures using hierarchically well-separated trees☆

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ABSTRACT

We consider the matching problem between two metric distributions where establishing a one-to-one matching of features may not always be possible. Although many-to-many graph matching techniques achieve the desired multi map between features, they ignore the spatial structure of the nodes. We propose a novel technique, *multilayer matching*, for solving the matching problem which utilizes both the individual node features and the clustering information of nodes. Our method uses the hierarchically well-separated trees (HSTs) to represent the metric distribution such that non-leaf nodes in the tree representation corresponds to a constellation of features in the original structure. By using HSTs in a linear programming setup, we obtain a matching between features through finding a mapping between non-leaf nodes among the two HSTs. We further provide a primal-dual approximation algorithm for the multilayer matching which runs several order of magnitudes faster while achieving comparable success rates. Application of the method to the image matching problem is also presented in the paper. Empirical evaluation of the method and its primal-dual extension on a set of recognition tests show the robustness and efficiency of the overall approach.

1. Introduction

Matching of metric distributions is a fundamental problem in computer science which has numerous real life applications such as image matching and object tracking. Since arbitrary metrics can be represented using graph structures, the problem can be reduced to graph matching, i.e., given two graphs G and H, find a mapping among vertices and edges of graphs. Exact graph matching is known to be computationally intractable. Specifically, *subgraph isomorphism* which asks whether G contains an induced subgraph H' that is isomorphic to H, is a problem known to be NP-complete [7]. Even a simpler special case of the problem, *graph isomorphism*, is neither known to be polynomial time solvable nor NP-complete. These facts led the research in the field to focus on *inexact matching* since early 80s [23].

Aside from inexact matching being a feasible way to attack the intractable problem of exact graph matching, it might even be desirable in some problem domains where a node in one graph should be matching to more than one node in the other graph. Image matching is an instance of this where image features are represented as graph nodes and relationships among features are structured with weighted edges. In this setup, features of two images representing the same object can have minor differences due to occlusion, noise, scaling, etc. As a result, establishing a mapping between sets of nodes amongst two graphs rather than establishing a one-to-one matching between individual nodes becomes necessary.

Metric embedding based methods are among the most popular approaches for tackling many-to-many matching problem [17]. Here, the objective is mapping data from a source space to a "simpler" target space while preserving the distances. Using embedding, a graph that is equipped with a metric can be approximated as a tree structure. It is well known that approximating the solution for many NP-hard problems in general metrics can be done in polynomial time once data is embedded into tree metrics. However, such embeddings tend to introduce distortion. A common technique for overcoming high distortion rates is *probabilistic approximation* method of Karp [16]. Utilizing probabilistic embedding, Bartal introduced the notion of *hierarchically well-separated trees* (HST) where edge weights on a path from root to leaves decrease by a constant factor in successive levels [3]. Embedding into HSTs improves the representational power of graphs especially in domains such as image matching since the internal nodes of the tree represents constellations of nodes of the original graph.

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Capturing the segmentation information at internal nodes of the tree along with the node features at its leaves make HSTs useful tools for inexact matching.

It is an underlying assumption that many-to-many matching algorithms use the features associated with individual nodes to establish the correspondences, while ignoring the underlying regions containing the spatial distribution of the matched features. In earlier work, we proposed a novel matching method, *multilayer matching*, that uses spatial distribution of nodes in addition to individual features to tackle the matching problem [22]. Motivated by the approach of Kleinberg and Tardos (K&T) for solving the *metric labeling problem*, our method uses optimization over HSTs [18]. We first embed two given graphs into separate HSTs and then match the nodes of the resulting trees at fixed layers by solving a linear program (LP) that is similar to the formulation of K&T. Obtaining a matching between the internal nodes of the HSTs makes it possible to take clustering structure of the features into account in addition to the individual image features. Utilizing the spatial distribution of features improves the running time of the matching task while accuracy is achieved by the use of feature vectors throughout the process.

Our method requiring to solve an LP makes the running time to be bound to number of variables and constraints of the LP formulation, and the efficiency of the LP solver. In this paper, we improve the performance of the method by presenting a primal-dual approximation algorithm for the multilayer matching problem [12]. Our primal-dual algorithm achieves a speed up by a factor up to 500x while obtaining comparable matching scores relative to baseline algorithms and other graph based methods from the pattern recognition literature. We demonstrate the utility of the method in the context of image matching. Experimental results indicate that the proposed algorithms perform well for standard datasets.

The rest of the document is organized as follows: Section 2 gives an overview of notations and definitions. In Section 3, we present the details of multilayer matching which is followed by a primal-dual approximation algorithm for the problem in Section 4. Next, we present the application of both methods to image matching problem in Section 5. Finally, Section 6 concludes the document.

2. Background and definitions

Over the last three decades, several methods were proposed for tackling the inexact graph matching problem including tabu search [26], error-correcting graph matching [5], and convex optimization formulations [1]. Graph edit distance based methods are among widely studied approaches for addressing noisy, many-to-many matching [2,4]. This approach performs local searches over the graph and transforms (part of) one graph into the other by minimal cost modifications such as inserting, deleting, substituting, or merging of nodes and edges. Using spectral graph theory, Caelli and Kosinov approached the problem as eigen-decomposition of adjacency matrix representation of the graph [6]. Zaslavskiy et al. formulated many-to-many graph matching as a discrete optimization problem and proposed an approximation algorithm based on continuous relaxation [27]. Another technique for dealing with the problem is by incorporating metric embedding methods. Although used in a different problem domain, pyramidal graph representation of Haxhimusa et al. can be contrasted to our multilayer matching approach [13].

Our approach to the inexact graph matching brings three major tools together from the domains of probabilistic embedding, operations research, and approximation algorithms. We present a summary of the tools in the rest of this section.

2.1. Metric embedding of graphs into HSTs

The term embedding refers to a mapping between two spaces [15]. Given a hard computational problem, the objective of embedding methods is to reduce the problem into another space where a tracktable (approximate) solution is possible. Thus, embedding techniques are commonly used for finding approximate solutions to NP-hard problems. A finite *metric space*[1] (P, d) can naturally be represented as a weighted graph $G = (V, E)$ with shortest path as the distance measure where points in P form the vertex set V and pairwise distances among points become the edge weights. Embedding of graphs into simpler structures is of particular interest since many problems involving graphs are known to be intractable. Simplifying graphs mainly involve operations such as removing edges that change the distance metric of the graph, removing or adding vertices, or changing weights of edges. This approach, however, introduces *distortion* on the distances defined over the graph.

In general, if not possible it is hard to find an isometric embedding between two arbitrary metric spaces. Therefore, it is important to find an embedding in which the distances between vertices of the destination metric are as close as possible to their counterparts in the source metric space. Trees are a feasible target space for many problems involving graphs since solving problems over trees is relatively easier compared to arbitrary graphs. Embedding of graphs into trees is a very challenging problem; even for the simple case of embedding an n-cycle into a tree. Karp introduced the notion of *probabilistic embedding* for overcoming this difficulty. Given a metric d defined over a finite space P, the main idea is to find a set S of simpler metrics defined over P which dominates d and guarantees the expected distortion of any edge to be small [16].

Uniform metrics are among the simplest tessellation spaces where all distances are regularly distributed across cells. Problems defined under such metrics can be easily solved by applying divide-and-conquer approaches which makes uniform metrics important from a computational point of view. Motivated by these observations, Bartal defined the notion of *hierarchically well separated trees (HST)* for viewing finite metric spaces as a uniform metric [3]. A k-HST is defined as a rooted weighted tree where edge weights from a node to each of its children are the same and decrease by a factor of at least k along any root to leaf path. Assuming that the maximum distance between any pair of points (diameter) in source space is Δ, source space is separated into clusters (sub-metrics) of diameter $\frac{\Delta}{k}$. The resulting clusters are then linked to the root as child nodes with edges of weight $\frac{\Delta}{2k}$. The relation between parent and child nodes continues recursively until the children nodes consist of single data elements. Bartal has shown the lower bound for distortion of embedding into HSTs to be $\Omega(\log n)$ and Fakcharoenphol et al. [11] introduced a deterministic embedding algorithm that achieves a tight distortion rate $(\Theta(\log n))$.

2.2. Metric labeling problem

One of the earliest algorithmic application of HST in general family of classification problems is known as *metric labeling* [18]. Given a set of objects $\mathcal{O}$ and a set of labels $\mathcal{L}$ with pairwise relationships defined among the elements of both sets, the goal is to assign a label to each object by minimizing a cost function involving both separation and assignment costs. Separation cost penalizes assigning loosely related labels to closely related objects while

[1] Given a set of points P, a mapping $d : P \times P \rightarrow R^+$ is called distance function if $\forall \tilde{}p, q, r \in P$ the following four conditions are satisfied: $d(p, q) = 0$ iff $p = q$, $d(p, q) \geq 0$, $d(p, q) = d(q, p)$, and $d(p, q) + d(q, r) \geq d(p, r)$. The pair (P, d) is called a *metric space* or a *metric*.