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High output dynamic range exponential function synthesizer

Cosmin Radu Popa

Faculty of Electronics, Telecommunications and Information Technology, Bucharest, Romania

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ABSTRACT

The paper presents two original high-accuracy exponential function synthesizers with wide output dynamic ranges. The improved accuracies of the proposed computational structures are obtained using new superior-order approximation functions. In order to additionally improve the circuits' output dynamic ranges, an original method based on proper variable changing will be used and implemented. As a result of the new proposed design methods, the performances of the exponential structures are only slightly affected by technological errors. The best original proposed architecture of the exponential synthesizer has an output dynamic range of 100 dB, for an approximation error smaller than ± 1 dB. The exponential circuits are designed for implementing in 0.18 µm CMOS technology and they present a low-voltage low-power operation – the minimal supply voltage is about 0.7 V, while the power consumption is smaller than 2 µW. The chip area is about 4.5 µm^2 for an implementation in 0.18 µm CMOS technology of the best proposed exponential circuit.

1. Introduction

A very important function used in signal processing, finding a multitude of applications in VLSI designs, is represented by the exponential function [1–35]. The previously reported methods for generating the exponential function using CMOS circuits can be grouped in two classes.

The first class of exponential function synthesizers uses the limited Taylor series expansion for obtaining the exponential function, having the main disadvantage of a relatively small output dynamic range (smaller than $30dB$) because of the limitations related to circuit complexity. Most of circuits [1–17] use the second-order approximation, only some of them [18,19] are based on the third-order approximation of the exponential function and present a smaller approximation error comparing with previous ones.

Other exponential computational structures [21–32] use different forms of approximation functions for generating the exponential function. The relation between the circuits' accuracy and the complexity of their CMOS implementation is favorable to the exponential function synthesizers using approximation functions, comparing with the exponential circuits based on limited Taylor series expansions. However, the disadvantage of a relatively large approximation error still represents an important problem of this second class of exponential function synthesizers, as a result of their small order of approximation. The most used in literature exponential function approximation functions are presented in (1) [21–27,34] - output dynamic range smaller than $26dB$, (2) [28–30,34] - output dynamic range of about $45 - 47dB$ (K and a being parameters), (3) [31,32,34] - output dynamic range smaller than $30dB$ (with a and b being parameters that can be controlled by the designer) or (4) [34], having an output dynamic range smaller than $40dB$. All of these previously reported exponential function synthesizers use second-order or third-order approximation functions, in order to reduce the silicon area of the circuit. The results is a relatively small output dynamic range of the exponential structures designed using these approximation functions, that drastically limits the area of applications for the developed exponential function synthesizers.

$$e_1(x) = \frac{1+x}{1-x} \cong exp(2x) \tag{1}$$

$$e_2(x) = \frac{K+(1+ax)^2}{K+(1-ax)^2} \tag{2}$$

$$e_3(x) = \frac{1+ax}{1+bx} \tag{3}$$

$$e_4(x) = \frac{1+ax}{1+bx} + cx \tag{4}$$

In this paper, it will be introduced and developed a new more accurately fourth-order approximation function, having the advantage of a reduced complexity and of a wide output dynamic range. The current-mode operation and the biasing in saturation of all MOS transistors that compose the proposed exponential function synthesizer are responsible for a very good frequency response of the circuit. The biasing in strong inversion (saturation region) of all MOS transistors is chosen in order to improve the frequency response of the developed exponential function synthesizer.

The utilization of the new proposed fourth-order high-accuracy approximation function for generating the exponential function allows to use approximately the same chip area for its CMOS implementation,

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comparing with previously reported third-order exponential function synthesizers, based on classical method of approximation. As the original developed exponential function synthesizer uses a higher-order (fourth-order) of approximation, its accuracy and, additionally, its output dynamic range will be much better comparing with previous similar works.

2. Theoretical analysis

The original proposed exponential function circuits generate the exponential function using new developed fourth-order approximation functions. As a result of the fourth-order approximation, the output dynamic ranges of the proposed circuits are strongly increased comparing with the previous reported similar circuits. In order to additionally increase the output dynamic range of the new designed circuits, an original variable changing will be used, allowing an important reduction of the effective range of the input variable.

The new proposed approximation methods allow extremely simple implementations in CMOS technology, using only two, respectively four current-mode squaring circuits, their total complexity being comparable with the complexity of previously reported exponential circuits with only second-order approximation and much poorer performances.

2.1. Synthesis and implementation of the first approximation function

In order to synthesize the general form of the approximation functions that will be used for generating the exponential function, some requirements must be fulfilled.

The most important conditions that are imposed to the approximation function are related to its capability to simulate with a very good accuracy the exponential function, correlated to the need for an extended dynamic output range for the designed computational structure. These performances have to be obtained using a reasonable complexity of the exponential function synthesizer. Usually, the increasing of the accuracy of approximation produces an important increasing of the complexity for the circuits that implement in CMOS technology the approximation function. From this point of view, a tradeoff between the acceptable complexity of the exponential function synthesizer and its overall accuracy must be habitually made. The new proposed method of synthesis the approximation function has the advantage of presenting an extremely low approximation error, this performance being obtained using a reasonable circuit complexity. This is possible by exclusively using in the circuit implementation of current-mode squaring circuits and of current mirror circuits, having relatively small intrinsic complexities in CMOS technology. Additionally, the original expression of the approximation function further reduces the circuit complexity. The current-mode operation of the proposed exponential function synthesizers and the independence of their output signal on technological parameters and on temperature variations contribute to a supplementary increasing of the circuits' accuracy, allowing to decrease the order of approximation, while maintaining, however, the accuracy of the computational structure.

The necessity of the exclusively use in the implementation of the exponential function synthesizers of MOS transistors biased in saturation region is imposed by the requirements for a good frequency response of the proposed exponential circuits. In the same situation, the current-mode operation strongly improves the frequency response of the proposed circuits.

The original proposed mathematical forms of the approximation function are synthesized starting from the general expression of the I_C output current of a current squaring circuit (because the squaring function is the most convenient choice for implementing in CMOS technology):

$$I_C = \frac{I_B^2}{aI_A} \tag{5}$$

The I_A and I_B currents are the input currents of the squaring circuit, while a represents a constant coefficient that depends on the particular implementation of the circuit. Using particular values of input currents: $I_A = I_O + bI_{IN}$ and $I_B = cI_O$, it is simple to generate, using a current squaring circuit with small complexity, an output current expressed as follows:

$$I_C = I_O \frac{c^2}{a} \frac{1}{1 + b\frac{I_{IN}}{I_O}} \tag{6}$$

In conclusion, new approximation functions will be obtained considering a finite number of primitive functions, having the general expressions derived from (6), $1/(1 + b_k x)$. The x variable is defined as the ratio between the input current and the reference current, $x = I_{IN}/I_O$. The number and the form of primitive functions that will represent the basis for synthesizing the approximation function is imposed by the necessity that its Taylor series must fulfilled the Taylor series of the exponential function. Evidently, the improving of the circuit accuracy can be made by increasing the number of primitive functions and, in consequence, the complexity of the exponential function synthesizer.

Because the absolute value of a superior-order term from a Taylor series expansion strongly decreases when the order of approximation increases, the fourth-order term from the Taylor series expansion of a continuous function will be much smaller (in absolute value) than the constant, linear, second-order and third-order terms from the same expansions. The additional consideration of the fifth-order term produces only a very small increasing of the approximation accuracy, but strongly increases the complexity of the computational circuits that will implement the approximation function. In consequence, taking into account a convenient tradeoff between the accuracy of approximation using a limited Taylor series expansion and the complexity of the implementation in CMOS technology of the approximation function, an optimal choice that permits to obtain a very good accuracy and a very large dynamic output range of the exponential function synthesizer circuit, using a reasonable circuit complexity will be based on a fourth-order approximation of the exponential function. The original proposed general form of the approximation function (which is able to fourth-order approximate a multitude of continuous mathematical functions) will be synthesized using two primitive functions and two additional linear terms. having the following general expression:

$$f_1\left(\frac{I_{IN}}{I_O}\right) = \frac{a_1}{1 + a_2\frac{I_{IN}}{I_O}} + \frac{a_3}{1 + \frac{I_{IN}}{I_O}} + a_4\frac{I_{IN}}{I_O} + a_5 \tag{7}$$

The reason for choosing the previous general form of the approximation function is related to its facile CMOS implementation and to its complete definition using a number of five coefficients (a_1 to a_5). In order to obtain the values of these coefficients (in the particular case of approximating the exponential function), the identity between the first five terms from the Taylor series of the approximation function and, respectively, of the exponential function must be considered (constant term, linear term and second, third and fourth-order terms). Solving the system imposed by the previous identities, it results the following expression of the fourth-order approximation function:

$$f_1\left(\frac{I_{IN}}{I_O}\right) = \frac{26}{5}\frac{1}{1 - \frac{5I_{IN}}{16I_O}} + \frac{1}{128}\frac{1}{1 + \frac{I_{IN}}{I_O}} - \frac{7I_{IN}}{11I_O} - \frac{21}{5} \tag{8}$$

The comparison between $f_1(x)$ first approximation function and $\exp(x)$ function is presented in Table 1, showing a wide decibel linear range of $33dB$, for a deviation from the exponential characteristic of $1dB$.

A graphical comparison between $f_1(x)$ approximation function and $\exp(x)$ function is presented in Fig. 1, illustrating a good fitting of $f_1(x)$ function with the exponential function.

The original proposed exponential function synthesizer permits a facile reconfiguration, the generation of any continuous mathematical