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Minimum-phase criterion on sampling time for sampled-data interval systems using genetic algorithms

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Abstract

In this paper, a genetic algorithm-based approach is proposed to determine a desired sampling-time range which guarantees minimum phase behaviour for the sampled-data system of an interval plant preceded by a zero-order hold (ZOH). Based on a worst-case analysis, the identification problem of the sampling-time range is first formulated as an optimization problem, which is subsequently solved under a GA-based framework incorporating two genetic algorithms. The first genetic algorithm searches both the uncertain plant parameters and sampling time to dynamically reduce the search range for locating the desired sampling-time boundaries based on verification results from the second genetic algorithm. As a result, the desired sampling-time range ensuring minimum phase behaviour of the sampled-data interval system can be evolutionarily obtained. Because of the time-consuming process that genetic algorithms generally exhibit, particularly the problem nature which requires undertaking a large number of evolution cycles, parallel computation for the proposed genetic algorithm is therefore proposed to accelerate the derivation process. Illustrated examples in this paper have demonstrated that the proposed GA-based approach is capable of accurately locating the boundaries of the desired sampling-time range.

Keywords: Minimum-phase; Genetic algorithms; Uncertain systems; Interval plant; Sampled-data systems; Discretization; Parallel computation

1. Introduction

It is well known that stability can be preserved when a stable continuous-time system is sampled by a zero-order hold (ZOH). However, there is no guarantee that zeros of the sampled system will always remain within the unit circle in the z-plane (i.e. minimum phase) [1]. In the design of linear control systems, the existence of unstable zeros makes it difficult to construct some control procedures, such as inverse systems, model matching systems, and model reference adaptive controllers [2]. Therefore, the unstable zeros, which limit the performance that can be achieved when controlling a system, should be avoided.

Unfortunately, there is no simple formula governing the mapping of zeros for the sampled system even in deterministic systems. An earlier discussion about zeros of sampled system [1] showed that if there are more than two excess poles in the continuous transfer function of the plant, the sampled system will be non-minimum phase (NMP) if the sampling frequency is sufficiently fast. Criteria, which guarantee that all zeros of sampled transfer functions are inside the unit circle, have been investigated [1–6]. They were all based on the observation of Nyquist plot of the sampled system. In Ref. [1], sufficient conditions for retaining stable zeros were given both in terms of continuous plant $G(s)$ and in terms of the Nyquist curve $G(j\omega)$, but they are quite restrictive. To determine the sampling-time range which ensures stable zeros, Fu and Dumont [4] provided an alternative way to obtain the critical sampling frequency by using a relay. This approach, however, is limited to cases when the continuous plant is a stable, minimum-phase system. To relieve the abovementioned restrictions, criteria for stability of zeros of sampled systems were proposed by extending the Nyquist method to unstable systems [5]. This method which graphically determines the stable zeros by examining the Nyquist plot, however, is limited to a specified sampling time T only. Moreover, if a low-pass plant is considered, the criteria, which guarantee minimum-phase, are represented by a non-linear inequality comprising sampling time T and coefficients of

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the plant. To determine the sampling-time range to ensure minimum-phase behaviour for the sampled system, this method [5] has no choice but to eventually resort to numerical methods to solve a non-linear equation of sampling time T. Based on symbolic manipulation, a symbolic approach [6] was proposed to determine the sampling-time range to ensure minimum phase behaviour. Although this method provides a simpler way in solving the minimum phase problem with fewer restrictions, it is workable only for deterministic systems.

As far as interval systems [17–19] are concerned, the problem of determining the sampling-time range to ensure minimum phase behaviour for the sampled system becomes extremely difficult because of the exponential nature [7] during the discretization process of the interval plant, where the coefficient functions of the sampled system involve serious and non-linear couplings of the sampling time T and the uncertain plant parameters. Due to the highly non-linear nature, it is extremely difficult to examine the behaviour of zeros of the sampled system in Nyquist plot as sampling time varies. Furthermore, because of the non-convexity inevitably involved in the determination of the sampling-time range to ensure minimum phase behaviour for the sampled system, conventional methods generally fail to solve this problem, particularly when the interval plant is of higher order. Generally speaking, there is still no systematic method so far to determine the sampling-time range to guarantee minimum phase behaviour for sampled-data interval systems.

Recent developments of evolutionary algorithms [8–13] have provided a promising alternative to address the above-mentioned problems and difficulties because of their capabilities of directed random search for global optimization [14,15]. This motivates the use of genetic algorithms to determine the sampling-time range to guarantee minimum phase behaviour for sampled-data interval systems. Based on a worst-case analysis, the identification problem of sampling-time range is first formulated as an optimization problem, which is subsequently solved by a proposed GA-based approach incorporating two genetic algorithms. The first genetic algorithm searches both the perturbation range of the uncertain plant parameters and sampling time to dynamically reduce the search range for locating the desired sampling-time boundaries ensuring minimum phase behaviour for the sampled-data interval system. The second genetic algorithms, on the other hand, searches only the perturbation range of the uncertain plant parameters to verify if minimum phase condition has been met for a specific sampling time T_c passed from the first genetic algorithm. As evolution continues, the search range for determining the sampling-time boundaries becomes narrower and narrower, and eventually converging to the desired solutions. By doing so, the desired sampling-time range ensuring minimum phase behaviour of the sampled-data interval system can be evolutionarily obtained. Because of the time-consuming process that genetic algorithms generally exhibit, particularly the problem nature which requires undertaking a large number of evolution cycles, parallel computation for the proposed evolutionary approach in a MATLAB-based working environment is therefore proposed to accelerate the derivation process.

This paper is organized as follows. Section 2 formulates the problem to identify the sampling-time range as an optimization problem. A proposed GA-based framework to determine the sampling-time range ensuring minimum phase is given in Section 3. Evolutionary schemes and parallel processing for the proposed GA-based approach to accelerate the derivation process are also given in this section. Illustrated examples are demonstrated in Section 4. Conclusions are drawn in Section 5.

2. Problem formulation

It is well known that mathematical models never exactly describe behaviours of a physical system. Environmental changes as well as production tolerances affect the values of the system's parameters [22]. The variations of the uncertain parameters generally do not follow any of the known probability distributions and are most often quantified in terms of bounds [23,24]. Hence the uncertain systems are usually represented by continuous-time uncertain models with interval parameters [22,23]. Simple as they might be, uncertain models in the form of interval systems have provided a convenient way in constructing mathematical models for physical systems, based on which practical design can be achieved for use in industry [25].

Consider a sampled-data system shown in Fig. 1, where the interval plant is given by

$$G_{\mathrm{p}}(s,\boldsymbol{a}^I,\boldsymbol{b}^I)=\frac{N(s,\boldsymbol{b}^I)}{D(s,\boldsymbol{a}^I)}=\frac{b_1^I s^{n-1}+b_2^I s^{n-2}+\cdots+b_{n-1}^I s+b_n^I}{a_0^I s^n+a_1^I s^{n-1}+a_2^I s^{n-2}+\cdots+a_{n-1}^I s+a_n^I},\tag{1}$$

where $N(s,\boldsymbol{b}^I)$ and $D(s,\boldsymbol{a}^I)$ are interval polynomials with coefficient vectors $\boldsymbol{b}^I=[b_1^I,b_2^I,\cdots,b_{n-1}^I,b_n^I]^T$ and $\boldsymbol{a}^I=[a_0^I,a_1^I,\cdots,a_{n-1}^I,a_n^I]^T$ consisted of uncertain interval parameters

$$b_j^I=[b_j^-,b_j^+]:=\{b_{jr}\in\Re|b_j^-\le b_{jr}\le b_j^+,\ j=1,2,3,\cdots,n\}\tag{2}$$

and

$$a_i^I=[a_i^-,a_i^+]:=\{a_{ir}\in\Re|a_i^-\le a_{ir}\le a_i^+,\ i=0,1,2,\cdots,n\},\tag{3}$$

respectively.

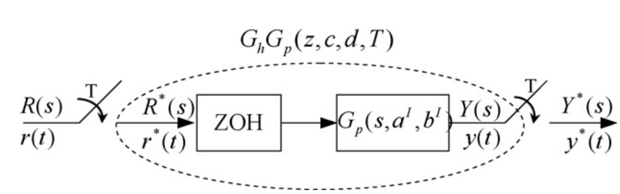

Fig. 1. A sampled plant with uncertain interval parameters.