Microelectronics Reliability xxx (2016) xxx–xxx

Contents lists available at ScienceDirect

Microelectronics Reliability

journal homepage: www.elsevier.com/locate/mr

A Wiener process model for accelerated degradation analysis considering measurement errors

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ARTICLE INFO

Article history:
Received 11 May 2016
Received in revised form 16 July 2016
Accepted 8 August 2016
Available online xxxx

Keywords:
Accelerated degradation analysis
Wiener process
Measurement errors
First hitting time
Reliability assessment

ABSTRACT

Accelerated degradation analysis plays an important role in assessing reliability and making maintenance schedule for highly reliable products with long lifetime. In practical engineering, degradation data, especially measured under accelerated condition, are often compounded and contaminated by measurement errors, which makes the analysis more challenging. Therefore, a Wiener process model simultaneously incorporating temporal variability, individual variation and measurement errors is proposed to analyze the accelerated degradation test (ADT). The explicit forms of the probability distribution function (PDF) and the cumulative distribution function (CDF) are derived based on the concept of first hitting time (FHT). Then, combining with the acceleration models, the maximum likelihood estimations (MLE) of the model parameters are obtained. Finally, a comprehensive simulation study involving two examples and a practical application are given to demonstrate the necessity and efficiency of the proposed model.

1. Introduction

Comparing with the traditional failure time analysis in terms of asymptotic efficiency, the degradation analysis has demonstrated a higher precision [1]. Thus, degradation analysis is commonly adopted in reliability assessment for today's highly reliable products and has been well developed over the years [2,3]. For many highly reliable products, there is always only a small amount of degradation can be inspected in a reasonable test time span at a use stress level. Insufficient degradation inspection inevitably results in inaccurate assessment of products' reliability, which will then lead to wrong decisions in maintenance scheduling. To settle this problem, ADT is frequently adopted as an effective solution.

Due to the attractive mathematical properties and physical interpretations, degradation models based on Wiener process have been extensively utilized to describe the accelerated degradation of products. A well-adopted form of the Wiener process in degradation analysis can be expressed as $X(t) = \beta\Lambda(t) + \sigma B(\Lambda(t))$, where β is the drift parameter representing the degradation rate and σ is the volatility parameter, $B(\cdot)$ denotes a standard Wiener process and $\Lambda(t)$ is defined as the transformed time scale.

Many researchers have applied this popular Wiener process model to conduct the ADT analysis. Park and Padgett [4] presented a class of accelerated degradation models including Wiener process, Gamma process and Gaussian process, and developed a statistical inference for both failure time data and degradation data. Lim et al. [5] developed an optimal constant-stress ADT (CSADT) plan, which assumed that the degradation characteristic followed a Wiener process. Liao and Tseng [6] provided an optimal step-stress ADT (SSADT) plan based on the assumption that the underlying degradation path followed a Wiener process through a time transformation. Tang et al. [7] developed an accelerated degradation process method with random effects for the nonlinear Wiener process. Similar research can be found in Liao and Elsayed [8], Pan and Balakrishnan [9], and Hua et al. [10].

In the degradation analysis, many real applications suggest that degradation of a batch of products is usually affected by three types of variability including temporal variability, individual variation and measurement errors [11–13]. The temporal variability is the inherent variation in the degradation for an individual item, the individual variation (also named product-to-product variation) describes the heterogeneity among the degradation paths of different units and the measurement errors are usually created during the measurement process in the degradation tests because of the imperfect instruments, procedures and environments. Consequently, to conduct a reasonable analysis, the three types of variability including temporal variability, individual variation and measurement errors have to be simultaneously considered in the modeling procedure. First, when a random process based model is adopted to depict degradation processes, the temporal variability is taken into account. Thus, the Wiener process based accelerated degradation model proposed in this paper has considered this variation. Meanwhile, the individual variation can be considered by

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http://dx.doi.org/10.1016/j.microrel.2016.08.004
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incorporating random effects into the Wiener process ADT model [7], where fixed effects capture the common characteristics while random effects capture the product-to- product differences. In the current study, the individual variation is also considered along the line of incorporating random effects. Regarding the measurement errors, however, they have not been considered in ADT analysis in the literature as far as the authors understand. Significant analysis and prediction errors may be produced when ignoring measurements errors in ADT modeling. Therefore, the first objective of the current study is to remedy this deficiency, and the necessity of incorporating measurement errors are further studied and illustrated via comparison. In addition, one should note that, uncertainty management has been discussed as a key problem in failure prognostics for an individual system under actual operating conditions [14,15]. Since the degradation analysis of a batch of products is focused on in the current study, the uncertainty management is not considered.

On the other hand, from the one-time-transformed Wiener process models, it should be noted that certain relationship, also can be called limitation, exists between mean and variance because of the only one time-transformed $\Lambda(t)$. Taking the linear transformed time scale situation (i.e., $\Lambda(t)$ is a linear function of test time) as an example, it is worth noticing that the time-transformed Wiener process will illustrate both linear trend $\beta\Lambda(t)$ and variance $\sigma^2\Lambda(t)$. Although for the commonly used mixed effects model situation $\beta\sim N(\mu,\kappa^2)$, the Wiener process will exhibit a linear mean $\mu\Lambda(t)$ and a quadratic variance $\kappa^2\Lambda^2(t)+\sigma^2\Lambda(t)$, this quadratic variance is not a generalized one because of the only one transformed time scale $\Lambda(t)$. In addition, it is especially noteworthy that when the degradation illustrates an increasing mean, the well-adopted Wiener process model and the mixed effect model will both show an increasing dispersity. In practical engineering, however, this is not always the case. For instance, the light emitting diodes (LED) deterioration data collected by Chaluvadi [16] via an accelerated test show an increasing mean and a decreasing variance.

To settle this problem, it is necessary to propose a generalized Wiener process model for ADT analysis. A generalized Wiener process model with two transformed time scales $\Lambda(t)$ and $\tau(t)$, i.e., $X(t)=\beta\Lambda(t)+\sigma B(\tau(t))$, has been investigated in [17,18]. This model can depict more degradation processes including the circumstance of decreasing dispersity the one-time-scale model cannot properly characterize. In real application, however, most researchers have restricted their attention to the case $\Lambda(t)=\tau(t)$, because it is difficult to obtain an explicit form of the lifetime distribution when $\Lambda(t)\neq\tau(t)$. Thus, motivated by practical needs, the second objective of this study is to develop a generalized Wiener process model with two time scale transformations to analyze the ADT. The constructed Wiener process model with measurement error includes the previously given popular form Wiener process (one time scale transformation model) and the other models as specified cases, and this makes the proposed approach attractive.

The remainder of the paper is organized as follows. Section 2 introduces the generalized Wiener process model with measurement errors for ADT analysis and the lifetime distribution is derived based on the FHT concept. Section 3 discusses the MLE for model parameters. In Section 4, the efficiency and reasonability of the established methodology is validated via a comprehensive Monte Carlo simulation study including two examples. In Section 5, the proposed approach is illustrated by a real application involving the ADT of LED and comparative results are given. A summary and conclusion is given in Section 6.

2. Accelerated degradation model

2.1. A generalized Wiener process model

Motivated by the practical needs, a generalized Wiener process degradation model with measurement errors can be defined as:

$$Y(t)=X(t)+\varepsilon=\beta\Lambda(t)+\sigma B(\tau(t))+\varepsilon \tag{1}$$

where $Y(t)$ and $X(t)$ denote the degradation inspections and the true degradation of the product's performance characteristic at time t, ε is the measurement error term which follows a normal distribution as $\varepsilon\sim N(0,\sigma_e^2)$, and $\Lambda(t)$ and $\tau(t)$ are transformed time scales. To capture the heterogeneity among the population, β is assumed to be random and follows a normal distribution $\beta\sim N(\mu_\beta,\sigma_\beta^2)$. This normally distributed random parameter β has been adopted by many studies [7,11,12]. For a preset observation time sequence $0<t_1<t_2<\cdots<t_n$, the measurement errors ε_i are assumed to be mutually independent of each other and of the true degradation $X_i=X(t_i), i=1,2,\dots,n$.

From the above description, it is clear that the proposed model denoted as M_0 encompasses the following models as limited cases that have been studied in the literature. Supposing $\Lambda(t)=\tau(t)=t$, model M_0 becomes the linear Wiener process model with measurement error, which has been studied by Peng et al. [13], Pan et al. [19], and Tang et al. [20]. If $\Lambda(t)=\tau(t)$, model M_0 turns to be the regular one time-transformed Wiener process model with measurement error, which has been studied by Ye et al. [11] and Whitmore [12]. Letting $\tau(t)=t$, model M_0 becomes the nonlinear Wiener process with measurement errors, which was utilized by Tang et al. [21] to predict the remaining useful lifetime. When $\sigma_e=0$, model M_0 becomes the Wiener process model not considering measurement errors proposed by Wang et al. [17,18].

2.2. Degradation modeling for ADT

In ADT analysis, the accelerated relationships are usually utilized to model the correlation between the degradation rate and the stress [2,3]. The common accelerated formulas include Arrhenius model, Eyring model and the power rule model. Thus, the relationship between the drift parameter β in Eq. (1) and the stress S can be expressed as:

$$\beta(S)=a\lambda(S;b) \tag{2}$$

where $a\sim N(\mu_a,\sigma_a^2)$ and $\beta\sim N(\mu_a\lambda(S;b),\sigma_a^2\lambda^2(S;b))$. Thereafter, $\lambda(S;b)$ can be obtained based on the acceleration relationship. For example, $\lambda(S;b)=\exp(-b/S)$ can be constructed according to the Arrhenius model.

The CSADT and SSADT are the two common types of ADT and have been widely studied in the literature, such as LED [22], OLED [23], SLD [24] and so on. Motivated by the practical needs, the generalized Wiener process degradation model with measurement errors given in Eq. (1) is utilized to model the CSADT and SSADT.

In a CSADT, let S_0 be a use stress level and $S_1<S_2,\dots,<S_L$ denote L higher stress levels. Suppose m_l units are tested under the stress level S_l, $l=1,2,\dots,L$. The degradation of each unit is measured at a prespecified test time sequence $t_1<t_2<,\dots,<t_n$. In this paper, for simplicity, letting $\beta_l=\beta(S_l)$, $\Lambda_i=\Lambda(t_i)$ and $\tau_i=\tau(t_i)$, then the degradation process under stress level S_l can be defined as:

$$Y(t|S_l)=X(t|S_l)+\varepsilon=\beta_l\Lambda+\sigma B(\tau)+\varepsilon \tag{3}$$

For a SSADT, suppose there are m units in test, and each unit is tested under L higher stress levels $S_1<S_2,\dots,<S_L$. The degradation of each item is measured at a prespecified test time sequence $t_1<t_2<,\dots,<t_{n_1}<t_{n_1+1}<,\dots,<t_{n_2}<,\dots<t_{n_{l-1}}<t_{n_{l-1}+1}<,\dots,<t_{n_l},\dots,<t_n$, where tnl denotes the time point when the stress level changes from S_l to S_{l+1}, $l=1,2,\dots,L-1$. Thus, the testing stress for each unit can be expressed as:

$$S=\begin{cases} S_1 & 0\le t\le t_{n_1} \\ S_2 & t_{n_1}<t\le t_{n_2} \\ \vdots & \vdots \\ S_L & t_{n_{L-1}}<t\le t_n \end{cases} \tag{4}$$