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Ant colony optimization with hill climbing for the bandwidth minimization problem

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Abstract

In this work, the problem of reducing the bandwidth of sparse matrices by permuting rows and columns is addressed and solved using a hybrid ant system to generate high-quality renumbering which is refined by a hill climbing local search heuristic. Computational experiments compare the algorithm with the well-known GPS algorithm, as well as recently proposed methods. These show the new approach to be as good as current best algorithms. In addition, an algorithm to randomly generate matrices with known optimal bandwidth is developed and used to evaluate results. Comparisons show that the new algorithm was able to find either the optimal solution or a solution very close to the optimal for most instances.

Keywords: Bandwidth minimization; Ant colony optimization; Hill climbing

1. Introduction

Ant colony optimization (ACO) [1] is a recent metaheuristic for solving difficult combinatorial optimization problems, where (artificial) ants repeatedly build solutions with the help of a common memory that can be modified. The common memory is motivated by pheromone trail laying in the behaviour of real ants. In each iteration, ants return to a queen after constructing solutions. Good solutions are chosen by the queen and the common memory updated so that ants will know what the good solutions are in subsequent iterations. ACO has been applied to problems such as the traveling salesman and quadratic assignment problems. In this paper, we propose a hybrid heuristic, ACO combined with hill climbing, to

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address the well-known combinatorial optimization problem—the bandwidth minimization problem. In our hybrid system, each ant performs hill climbing before returning to queen. Computational experiments show that ACO with hill climbing is effective and comparable to recently proposed metaheuristics for the problem, including Tabu Search and GRASP with path relinking, in solution quality and running times.

The bandwidth minimization problem (BMP) is defined as follows: given a graph $G = (V, E)$, where V is the vertex set and E is the edge set, find a permutation p of V that minimizes $\max(|p(i) - p(j)|)$ where $(i, j) \in E$ (i.e. vertices i and j are connected), or equivalently, the length of the longest edge when the vertices are ordered on a line. In the context of matrices, given a matrix $A = [a_{ij}]$, the problem consists of finding a permutation of rows and columns that keeps all the nonzero elements of A in a band that is as close as possible to the main diagonal. The graph bandwidth minimization problem and the matrix bandwidth minimization problem are interchangeable using A as the incidence matrix of G.

The BMP received much attention in the 1950s resulting from computations on sparse matrices. Such matrices arise from solving linear systems where Gaussian elimination can be performed in $O(nb^2)$ time on matrices of bandwidth b. This is a large improvement over the general $O(n^3)$-algorithm when $b \ll n$. Bandwidth minimization is also used in other applications including problems in finite element methods for approximating solutions of partial differential equations, large-scale power transmissions systems, circuit design, hypertext layout, chemical kinetics and numerical geophysics.

Due to its wide practical use in engineering applications and other areas of science, the BMP has been studied extensively. In 1976, Papadimitriou proved the problem to be **NP**-complete [2]. A number of approximation algorithms have been proposed since 1960s, most of which only work well on particular kinds of graphs. In 1998, Unger [3] showed that, for any constant k, it is **NP**-complete to find any k-approximation, even for a class of restricted trees (caterpillars). The approximation of bandwidth between a constant and a poly-logarithmic factor remains an open problem. An alternative approach to the BMP is to use heuristics, many of which have been developed to solve the problem. The most well-known ones are the CutHill–McKee algorithm (CM) [4], King's algorithm [5], and the Gibbs–Poole–Stockmeyer algorithms (GPS) [6]. Most of these are based on a level structure of the graph, which partitions its vertices into levels $L_0, \ldots, L_n$ such that the endpoints of every edge in the graph are either in the same level L_i or in two consecutive levels L_i and L_{i+1}. By labeling the vertices level by level, a relatively good permutation giving a small bandwidth is obtained. Such algorithms based on a level structure are often grouped in the CM family. The most recent CM variant is the WBRA algorithm [7]. Metaheuristics used include Tabu Search [7,8] and GRASP with path relinking [9], both of which outperform the level structure algorithms in terms of solution quality, in are able to do this in reasonable amounts of time.

The following notation is used throughout the paper.

Given a graph $G = (V, E)$, we use the following notation in this work.

f – A labeling of vertices of G. Assuming $|V| = n$, f assigns the integers $\{1, 2, \ldots, n\}$ to the vertices of G. Different vertices will have different labels.
$f(v)$ – The label of vertex v under labeling scheme f.
$B_f(v)$ – The bandwidth of a vertex v under labeling scheme f. $B_f(v) = \max\{|f(v) - f(u)| : u \in N(v)\}$ where $N(v)$ is the set of vertices adjacent to v.
$B_f(G)$ – The bandwidth of the graph G with respect to f. $B_f(G) = \max\{B_f(v) : v \in V\}$.

The paper is organized as follows: in the next section, we present the basic structure of the new approach; in Sections 3 and 4, we explain how ACO and hill climbing are applied to the BMP. In Section 5, computational results are provided comparing the new approach with other methods, and in Section 6, we conclude by suggesting possible directions for future work.

2. Structure of the algorithm

A metaheuristic based on ant behavior can be described by a set of processes that are coordinated