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Denoising 3-D magnitude magnetic resonance images based on weighted nuclear norm minimization

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ABSTRACT

A new denoising algorithm based on low-rank matrix approximation (LRMA) with regularization of weighted nuclear norm minimization (WNNM) is proposed to remove Rician noise of magnetic resonance (MR) images. This technique simply groups similar non-local cubic blocks from noisy 3D MR data into a patch matrix with each block lexicographically vectorizing to be as a column, calculates the singular value decomposition (SVD) on this matrix, then the closed-form solution of LRMA is achieved by hard-thresholding different singular values with a different threshold. The denoised blocks are obtained from this estimate of the low-rank matrix, and the final estimate of the whole noise-free MR data is built up by aggregating all the denoised exemplar blocks that are overlapped each other. To further improve the denoising performance of the WNNM algorithm, we first realize the above denoising procedure in a two-iteration regularization framework, and then a simple non local means (NLM) filter based on single-pixel patch is utilized to reduce the intensity jumping at the homogeneous area. The proposed denoising algorithm was compared with related state-of-the-art methods and produced very competitive results over synthetic and real 3D MR data.

1. Introduction

Magnetic resonance imaging (MRI) has been widely used for clinical investigation and disease diagnosis. However, due to random fluctuations in the patient body or receiving coil electronics, the acquired image is often suffered from noises. Such noises may affect visual inspection and other image post-processing tasks such as segmentation or registration. Therefore, noise reduction is an important task in MR image analysis. Different from the denoising problem in natural images where noise is often modeled as Gaussian, noise in magnitude MR images is assumed as following a Rician distribution [1]. This noise model is derived by assuming the existence of zero-mean Gaussian noise with same variance in both real and imaginary channels of the complex raw data represented in the frequency domain (*k*-space). Rician noise is not zero-mean, and the mean depends on the local intensity in the image. Specifically, it follows a Rayleigh distribution in low-intensity regions and a Gaussian distribution in high-intensity regions. The signal-dependent nature in MR images severely decreases the image contrast and also poses a great challenge for noise reduction in these images.

Numerous denoising methods tailored specially for the Rician noise have been proposed in the past few decades. For example, gradient-based filters, including anisotropic diffusion (AD) filters [2–4] and filters using total variation regularization [5–7], are able to remove noise while respecting important image structures. Filters based on different neighborhood statistics were also widely studied in this arena, such as the linear minimum mean square error (LMMSE) estimator [8,9], the estimation of Rician distribution with expectation-maximum formulations [10,11], and the Bayesian estimation using higher-order neighborhood statistics as priors [12]. These classical filters can usually provide a good performance for removing Rician noise, however, they often suffer from loss of small details and generate piece-wise constant due to the various underlying assumptions of smoothness or neighborhood stationarity.

Wavelet and other transform-based noise reduction techniques also play an important role in the arena of MR image denoising. Nowak [1] applied a wavelet-based wiener-filter on the squared magnitude MR images. The work in Ref. [13] utilized the complex wavelet directly on the raw data of MR images that were obtained in the complex *k*-space. Bao and Zang [14] proposed a multiscale

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wavelet denoising method for MR images, which exploited the interscale dependencies of wavelet coefficients. More recently, Fathi and Naghsh-Nilchi [15] proposed a statistically optimum adaptive wavelet packet (WP) thresholding function for image denoising, and its application to medical images outperformed some of the best state-of-the-art wavelet-based denoising techniques. Besides wavelet, other fixed-basis transforms such as curvelet [16] and contourlet [17] have also been proposed for MR image denoising during the past decades. Although these fixed-basis transforms have demonstrated its efficiency in denoising Rician noise, it has the drawback of introducing many visual artifacts in the denoising output since only one fixed basis cannot well represent all the local structures existed in the MR images. To overcome this problem, transforms using data-adaptive basis such as the principal components analysis (PCA) [18], singular value decomposition (SVD) [19], or basis by dictionary learning [20] have also been adapted for MR image denoising. In fact, the common idea shared by all these transform-based denoising methods is that signals in a local window can be sparsely represented on certain basis. Thus, denoising can be realized by some thresholding technique to discard those noise-dominant components or to approximate the noise-free components in the transform domain. However, how to design a data-adaptive basis and threshold is still a great challenge.

Contrary to the effort of developing denoising methods based on local information or *domain*-Markovian assumption [21], another line of denoising algorithms [22–26] that exploits the non-local similarity or redundancy contained within the image has also achieved remarkable performances. The original idea of non-local similarity was traced back to the non-local means filter [27,28], and its adaptations to MRI images have been studied in numerous works [22–25]. Furthermore, during the past several years, the combination of signal sparseness and non-local similarity has led to a flurry of algorithms with remarkable performances in denoising natural images with Gaussian noises, such as block matching with 3D filtering (BM3D) [29], PCA-based denoising method with local pixel grouping (LPG-PCA) [30] and learned simultaneous sparse coding (LSSC) [31]. Similar efforts [19,26,32] have also been devoted to the denoising of MRI images, and state-of-art performances were often obtained in these works.

More recently, low-rank matrix approximation (LRMA), which aims to recover the underlying low-rank matrix from its degraded observation, has attracted more and more attentions in computer vision and machine learning. There has emerged an explosion of research in its algorithms and applications, such as face recognition [33], background modeling [34], video denoising [35], color image denoising [36] and image restoration [37]. It is well known that the matrix formed by similar nonlocal image patches has a low-rank structure in its nature. Therefore, LRMA provides an effective approach for modeling the nonlocal similarities existed in many images.

The present paper aims to investigate and improve the application of LRMA to the denoising of MR volume data. As a recently proposed approach, the LRMA based on weighted nuclear norm minimization (WNNM) was applied to the matrix formed by similar noisy cubic patches. The denoised patches were obtained from the low-rank reconstruction of the input matrix, and the estimation of the whole noise-free MR volume data was generated by aggregating all the denoised patches with a weighted averaging manner. For a better denoising performance, the above LRMA-based algorithm was realized within a two-stage iterative regularization framework. The basic idea of iterative regularization is to add some filtered noise back to the denoised image, thus some weaker signals that are removed as noise during the last iteration can be identified in the current iteration. Finally, we also realized a non-local means filter to reduce the intensity unevenness at the homogeneous area using the denoised image by LRMA as a reference image.

2. Materials and methods

2.1. LRMA-based denoising

The basic idea of LRMA-based denoising approach is to estimate true noise-free image patches by low-rank modeling of non-local similarities associated with them. Specifically, the image patches are grouped by block matching, such that the patches in each group share similar underlying image structures. Thus, by vectorizing an image patch into a column, all these similar patches are stacked into a matrix, which forms a noisy version of an approximate low-rank matrix. Accordingly, recovery of original image patches is cast as the problem of LRMA.

Given a noisy input patch matrix $Y \in \Re^{m \times n}$, the goal of LRMA is to find out the latent low-rank matrix to approximate the original clean patch matrix X with the following model

$$Y = X + N \quad (1)$$

where N denotes the noise matrix, which is often modeled as zero mean Gaussian noise with i.i.d entries drawn from $\mathcal{N}(0, \sigma^2)$.

To derive the low-rank matrix X from noisy observation Y, a popular approach is to impose an additional rank constraint upon X since direct rank minimization is NP hard. The imposed constraint on the estimated matrix X is often known as nuclear norm minimization (NNM), which is defined as the sum of its singular values, i.e., $\left\|X\right\|_* = \sum_i \lambda_i(X)$, where $\lambda_i(X)$ denotes the ith singular value of X. Thus, the problem of LRMA for Eq. (1) can be formulated as

$$\hat{X} = argmin_X \frac{1}{2}\left\|Y - X\right\|_F^2 + \tau\left\|X\right\|_* \quad (2)$$

where the first term is F-norm data fidelity function and τ is the regularization parameter. It has been shown in [38] that the solution to Eq. (2) can be easily obtained by imposing a soft-thresholding operation on the singular values of the observation matrix. That is

$$\hat{X} = US_\tau(\sum)V^T \quad (3)$$

where $Y = U\sum V^T$ is the SVD of Y and $S_\tau(\sum)$ is the soft-thresholding function on diagonal matrix $\sum = diag(\{\lambda_i(X)\}_{1 \le i \le r})$ with parameter τ. For each diagonal element $\lambda_i(X)$ in $\sum$, there is

$$S_\tau(\sum) = diag(\{\{\lambda_i(X) - \tau\}_+\}_{1 \le i \le r}) \quad (4)$$

where $(\xi)_+ = \max(\xi, 0)$. Though the estimation strategy shown in Eq. (4) has achieved great success in many applications, the traditional NNM-based LRMA also has some limitations. In this approach, all the singular values are shrunk toward zero with the same value τ. But as we know, different singular value of a practical matrix often conveys different amount of information, a larger value may represent more important structural information and should be shrunk with a smaller degree. It has been shown that the equal shrinking scheme in Eq. (3) is sub-optimal to recover the underlying signal matrix in the denoising problem [39].

In our denoising algorithm, we adopted a LRMA approach based on weighted nuclear norm minimization (WNNM). The weighted nuclear norm is defined by assigning a non-negative weight ω_i to each singular value $\lambda_i(X)$, that is

$$\left\|X\right\|_{\omega,*} = \sum_i \omega_i \lambda_i(X) \quad (5)$$