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Some new results of designing an IIR filter with colored noise for signal processing ☆

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ABSTRACT

The design of adaptive finite impulse response filters is a linear optimization problem and the design of adaptive infinite impulse response (IIR) filters in the presence of observation noise is a nonlinear optimization problem. This paper considers the parameter estimation issues of an infinite impulse response (IIR) filter with colored noise which is treated as an autoregressive process. The key is to investigate novel estimation methods of an IIR filter with an autoregressive disturbance noise from the viewpoint of the observation data filtering. Firstly, we simply give the least mean square (LMS) algorithm for an IIR filter with autoregressive noise and derive a multi-innovation LMS (MI-LMS) algorithm for improving the parameter estimation accuracy. Secondly, we present a data filtering based LMS algorithm and a data filtering based MI-LMS algorithm for further improving the parameter estimation accuracy. The theoretical analyses show that the proposed algorithms are convergent and the simulation results indicate that the MI-LMS algorithm and the data filtering based MI-LMS algorithm are superior to the LMS algorithm and the data filtering based LMS algorithm in accuracy, respectively. The proposed methods in this paper have been extended to an IIR filter with autoregressive moving average noise. Finally, two simulation examples test the performances of the proposed algorithms.

1. Introduction

Modeling and parameter estimation are important for model-based control strategies [1–3]. Time series analysis involves three basic models – autoregressive (AR), moving average (MA) and autoregressive moving average (ARMA) models. For the deterministic cases, the MA models are the finite impulse response (FIR) models [4,5] and the ARMA models are the infinite impulse response (IIR) models [6]. Many filtering and estimation methods have been developed for linear systems and nonlinear systems [7]. For example, the typical ones include the least mean square (LMS) algorithms and the recursive least squares (RLS) algorithms [8] and others include the Levinson–Durbin algorithm for determining the order and parameters of AR processes, the least squares based iterative estimation algorithm for MA processes [9], the least squares algorithm for ARMA processes [10].

The parameter estimation of ARMA models is intrinsically nonlinear and thus is more complex compared with AR and MA models [11]. For ARMA processes, a recursive least squares algorithm and a stochastic gradient algorithm have been presented for nonstationary processes and the convergence results of the algorithms have been established by means of the martingale convergence theorem [10]. The key is using the estimation residuals in place of the unmeasurable noise terms in the information vectors for dealing with moving average noise. This method has been used in the parameter identification for linear-in-parameters output error moving average systems [12], for bilinear systems with moving average noise and autoregressive moving average [13–15], and for output nonlinear autoregressive moving average systems [16].

The FIR models and IIR models (or filters) play a great role in signal processing and adaptive filtering and have been widely applied to many areas such as speech signals and data communication [17]. In essence, determining the optimal parameters for FIR filter structures is a linear problem but that for IIR filter structures is a nonlinear problem [11]. This motivates us to investigate novel parameter estimation methods of an IIR filter with an autoregressive disturbance noise using the observation data filtering.

The adaptive filtering and parameter estimation algorithms have wide applications in state estimation and prediction [18,19],

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optimization [20–22], signal processing [23,24] and information fusion [25,26]. Wang presented a filtering and auxiliary model based least squares iterative identification algorithm for output error moving average systems [27]; Ding et al. used the data filtering technique to derive a decomposition based least squares iterative identification algorithm for multivariate pseudo-linear ARMA systems [28]; Pan et al. studied a filtering based multi-innovation extended stochastic gradient algorithm for multivariable controlled autoregressive moving average systems [29]; Li and Shi designed the robust H-infinity filtering for nonlinear stochastic systems with uncertainties and random delays modeled by Markov chains [30]; Wang and Tang used the data filtering technique to derive several gradient based iterative parameter estimation methods for a class of output nonlinear stochastic systems [31]. The mentioned filtering based algorithms can give more accurate parameter estimates compared with the recursive or iterative based algorithms. However, most of the work uses the simulation experiments to test the effectiveness of the presented methods but does not illustrate their convergence performance from the view of theoretical analyses.

The convergence of the LMS and least squares algorithms for designing the FIR and IIR filters is important and has fully been investigated under white noise disturbances in the fields of adaptive signal processing and parameter estimation. In this aspect, the performance of the norm constraint least mean square algorithm has been analyzed [32]; the convergence analysis of the stochastic gradient algorithm and the recursive least squares algorithm have been carried on for estimating the parameters of nonstationary ARMA processes [10] by means of the martingale convergence theorem in [33]. However, to the best of our knowledge, the convergence of the IIR filtering algorithms has not been *fully* investigated, especially the IIR filtering algorithms with AR noise, MA noise or ARMA noise. The theoretical difficulty for performing the convergence analysis is that the identification of the noisy IIR filter involves not only the unknown parameters of the noise model but also the unmeasurable related noise. This paper resolves partially the long standing open problem of the global convergence and parameter estimation consistency of the adaptive IIR filtering algorithms in the presence of additive AR noise. It is worth pointing out that the methods proposed in this paper can be extended to study the convergence of IIR filtering algorithms with MA noise or ARMA noise.

Some adaptive filtering algorithms adopt the idea of the innovation modification that the current estimate is equal to the previous estimate plus a modified term, e.g., the LMS algorithm and the recursive least squares algorithm. According to the innovation modification technique, the multi-innovation identification theory has been developed for improving parameter estimation accuracy [34–36]. Compared with the conventional stochastic gradient (SG) algorithms [33,37], the multi-innovation SG algorithm has faster convergence rates and its basic idea is expanding the innovation length and fully using the useful information from observation data [38]. Recently, the multi-innovation identification theory has been used to solve the identification problem of linear systems and nonlinear systems [39]. On the basis of the work in [40,41], this paper applies the multi-innovation theory and the filtering technique to study novel parameter estimation methods of an IIR filter with autoregressive disturbance noise and the main contributions are as follows.

- By means of the gradient search and according to the multi-innovation theory, this paper gives a least mean square (LMS) algorithm for an IIR filter with autoregressive noise and derives a multi-innovation LMS (MI-LMS) algorithm for improving convergence rates.
- From the viewpoint of the observation data filtering, this paper derives a data filtering based LMS algorithm and a data filtering based MI-LMS algorithm for further improving the parameter estimation accuracy.
- With the help of the stochastic process theory, this paper analyzes the convergence of the proposed algorithms and indicates that the MI-LMS algorithm and the data filtering based MI-LMS algorithm are superior to the LMS algorithm and the data filtering based LMS algorithm in accuracy, respectively.
- The proposed data filtering based methods in this paper have been extended to an IIR filter with autoregressive moving average noise. The simulation examples test the performances of the proposed algorithms.

The structure of this paper is organized as follows. Section 2 describes the problem formulation. Section 3 gives an LMS algorithm for an IIR filter with AR noise and studies its convergence. Section 4 derives a multi-innovation LMS algorithm. Sections 5 and 6 derive a data filtering based LMS algorithm and a data filtering based MI-LMS algorithm, respectively. Section 7 extends the data filtering based MI-LMS algorithm to IIR filters with ARMA noise. Section 8 provides two examples to verify the effectiveness of the proposed algorithms. Finally, some concluding remarks are offered in Section 9.

2. The problem formulation

Let us define some notation for convenience. The symbol $\boldsymbol{I}_n$ represents an identity matrix of size $n \times n$; $\mathbf{1}_n$ represents an n-dimensional column vector whose entries are all unity. The superscript T represents the matrix/vector transpose. Let $\hat{\boldsymbol{\theta}}(k)$ and $\hat{\boldsymbol{c}}(k)$ be the estimates of the parameter vectors $\boldsymbol{\theta}$ and $\boldsymbol{c}$ at time k. The acronym a.s. denotes "almost surely" and the acronym m.s. denotes "mean square".

For the unknown IIR system with its disturbance being additive white noise, there are many approaches for discussing its filtering and parameter estimation problems. This paper considers an unknown IIR system and assumes that the disturbance is colored noise. We use an equation-error adaptive IIR filter to estimate the IIR system from the input–output observations. The IIR parameter vector of the unknown system is described by $\boldsymbol{\theta} := [a_1,\ a_2,\ \cdots,\ a_{n_a},\ b_0,\ b_1,\ \cdots,\ b_{n_b}]^{\mathrm{T}}$. The desired output $y(k)$ is given by

$$y(k) = \boldsymbol{\varphi}^{\mathrm{T}}(k)\boldsymbol{\theta} + w(k), \tag{1}$$

where $\boldsymbol{\varphi}(k) := [-y(k-1), -y(k-2), \cdots, -y(k-n_a), u(k), u(k-1), \cdots, u(k-n_b)]^{\mathrm{T}} \in \mathbb{R}^{n_0}$, $n_0 := n_a + n_b + 1$. Suppose that the AR order n_a and the MA order n_b are known. In general, the disturbance $w(k)$ is a correlated noise and can be simply an autoregressive moving average (ARMA) process:

$$w(k) = \frac{E(z)}{F(z)} v(k),$$

$\{v(k)\}$ is stochastic white noise with zero mean and unknown variance σ^2, $E(z)$ and $F(z)$ are the monic polynomials in the unit backward shift operator z^{-1} ($z^{-1}y(z) = y(z-1)$). Using the long division, we have

$$\frac{F(z)}{E(z)} = 1 + c_1 z^{-1} + c_2 z^{-2} + \cdots + c_i z^{-i} + \cdots$$

When $E(z)$ is stable, the coefficient c_i goes to zero as i increases. Thus for sufficient large n_c, $C(z) := 1 + c_1 z^{-1} + c_2 z^{-2} + \cdots + c_{n_c} z^{-n_c}$ can approximate $\frac{F(z)}{E(z)}$ well. In this case, we have

$$w(k) = \frac{1}{C(z)} v(k) \tag{2}$$

namely, an autoregressive noise process.