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Review of the quasi-maximum likelihood estimator for polynomial phase signals

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ABSTRACT

Several papers have been published recently on the quasi-maximum likelihood (QML) approach for the polynomial phase signals (PPS) parameter estimation. The QML is powerful alternative to phase differentiation (PD) based techniques. It achieves the Cramer–Rao lower bound (CRLB) for low SNR values even for high-order PPS. The QML is generalized for signals with non-polynomial modulation and can work for impulsive noise environment. Since it is based on the linear time–frequency (TF) representation, short-time Fourier transform (STFT), it is suitable for estimation of multicomponent (mc) PPS. The goal of this review article is to summarize all developments of this tool. The QML versions for both one-dimensional (1-D) and two-dimensional (2-D) signals, for known and unknown order of phase polynomial, and for non-polynomial frequency modulated (FM) signals, are presented. In addition, estimation of mc and aliased PPSs sampled bellow the sampling theorem rate are also considered as well as application of the random sample consensus (RANSAC) algorithm for further reduction of the signal-to-noise ratio (SNR) threshold. Generalization of the QML approach to the direction-of-arrival (DOA) of the PPS signals impinging on the sensor array network with some open issues in the QML theory and applications are also addressed. Several examples demonstrate accuracy and advantages of the reviewed techniques.

1. Introduction

Engineers in many fields often encounter with non-stationary frequency modulated (FM) signals that are found in biological, speech and music signals, wireless communications and radars, combustion knocks in car engines, medicine, and dispersive seismic signals, etc. Majority of recent articles reviewed in [1] has been devoted to analysis, estimation and processing of FM signals in the radar signal processing. For example, processing of recordings from over-the-horizon radar systems (OTHR) is challenging. These signals have complicated FM signatures, and they are corrupted by high noise, clutter and other disturbances [2,3]. Precise estimation of the OTHR recordings are important for detection of low flying aircrafts used in illegal activities. High resolution and precise estimation of FM signal has equally important application in other types of radar systems (SAR – synthetic aperture radar, ISAR – inverse SAR) [4–7]. The micro-Doppler effect estimation that is commonly modeled as the sinusoidal FM signal is crucial for both, its elimination from radar images and estimating various important features of radar targets that can be recognized from such signals [8–11]. Medical applications are in analysis of electroencephalogram (EEG) signals [12,13], and other [14]. Recently described challenge is estimation of signals from the linear electromagnetic encoders [15]. It has been shown that these signals require complicated modeling with a compound/combined FM model [16]. Other applications are reviewed in [1,17–21]. Nowadays, different direction-of-arrival (DOA) estimation algorithms for FM signals in sensor array networks are proposed able to produce highly accurate results for challenging signal mixtures [22].

Several different frameworks are proposed for analysis and estimation of FM signals. The time–frequency (TF) and time-scale (TS) methods and transforms are applied for analysis of FM and other nonstationary signals [20,21,23]. The TF and TS transforms are applied with the smallest available information about the signal of interest. They provide information about signal modulation, position in the TF/TS plane, strength and mutual relationship of signal components in the TF/TS domains. Based on the TF/TS transform signal components can be localized in the considered plane followed by the extraction of their parameters, filtering, processing using nonparametric or parametric estimators. The other frame-

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Table 1
Pros and cons of existing PPS estimators.

Estimators	Advantages	Disadvantages
ML estimators	Accurate; can work with mc-PPSs	Calculation burden with multidimensional search
PD estimators	Complexity; can work with mc-PPSs	Accuracy; limited in dealing mc-PPS
PU estimators	Low complexity [31]; high accuracy [32]	Low accuracy [31]; high complexity [32], cannot work with mc PPSs

work is related to non-parametric estimation, filtering and signal enhancement techniques with unknown signal model [24–29]. Non-parametric tools estimate signal parameters in each instant using some limited knowledge on the signal like, for example, part of the TF plane occupied by the signal, number and mutual relationship between components, or the fact that considered signal is mono-component with slow-varying amplitude with respect to its phase. Parametric estimators assume some particular model of signal phase, instantaneous frequency (IF), and/or amplitude and usually estimate signals based on a small number of parameters [1,30]. For parametric estimation of the FM signals, commonly polynomial phase signal (PPS) model is adopted [30–41]. The PPS model has constant or slowly varying amplitude and phase function represented by a polynomial. More complicated model is the multicomponent (mc) PPS representing additive mixture of several different PPSs. The practical PPS recording are corrupted by a noise. In research articles the stationary additive white Gaussian noise is the most commonly considered while in practice other types of disturbances also appear. The goal of the PPS estimators is to estimate phase parameters (coefficients of the phase polynomial) and amplitude from noisy recordings as accurately as possible and with as small as possible calculation complexity. In addition, it is important that developed tools are robust to changes in underlying assumptions or that can be modified to meet different circumstances. Developments in these three frameworks are often considered separately but in the reviewed technique they are closely related.

There are three main groups of the PPS estimators:

- Maximum likelihood (ML) estimators;
- Phase differentiation (PD) techniques; and
- Phase unwrapping (PU) estimators.

Detailed review of the ML and PD estimators can be found in [1, 30]. The PU techniques were known to be sensitive to the noise influence [31], while recent advance in the field [32] have improved accuracy that is paid by high calculation demands. The PU estimators are not able to deal with mc PPSs that are important for almost all practical applications. Pros and cons of these estimators are summarized in Table 1. Note that some other PPS estimators exist too, such as those based on the constant modulus algorithm [33], but their application is limited to narrowband signals or to signals affected by moderate noise.

For a long time, scientists and researchers dealing with the parameter estimation of FM signals avoided consideration of high-order PPS models due to lack of algorithms accurately estimating their parameters. PD-based techniques were the most popular PPS estimators. The high-order ambiguity function (HAF) was assumed as simple tool from this class giving state-of-the-art results [30, 34–41]. The HAF employs the PD to reduce order of the phase polynomial to 1 (complex sinusoidal) followed by estimation of the highest-order coefficient using maximization of the Fourier transform. The other coefficients are estimated by repeating the same procedure on the signal with removed previously estimated highest-order phase coefficients. However, these techniques have numerous shortcomings. Each employed PD significantly increases estimation error, lower order coefficients exhibits propagation of error from the highest order coefficients, interference (i.e., cross-terms introduced by nonlinearity in the PD evaluation process) makes estimation of mc PPS difficult. In the last two decades, this technique has been improved resulting with several approaches. Among them, the most popular is the product HAF (PHAF) [42] proposed for removing cross-terms in mc PPSs but it also improves overall robustness to the noise influence. Next important development in the PPS estimation is the cubic phase function (CPF) [43–49]. This technique reduces number of PDs with respect to the HAF that are the main source of noise influence increase in all related estimators. However, the CPF is able to estimate PPSs up to the third-order and, in the last couple of years, there are numerous CPF extensions related to both estimation of higher-order and multidimensional PPSs [48,50,51]. Some developments for the mc PPS can be found in [19,42,52–59].

The quasi-maximum likelihood (QML) approach has been proposed recently serving as powerful alternative to the PD-based techniques [60–64]. In the maximum likelihood (ML) estimation, phase parameters are obtained by performing a multidimensional search over entire parametric space. Due to high computational burden, the ML is limited to low-order PPSs. As opposite to the ML, in the QML, search is performed over single parameter, i.e., window length in the short time-Fourier transform (STFT). The STFT is linear TF representation robust to noise influence but biased for PPS with order higher than two [65–69]. Due to the bias, this transform has been kept out of the research attention in the parametric estimation field for a long time. The QML development has been motivated by the O'Shea refinement strategy [70]. This strategy is developed for improving results of relatively accurate PD-based estimators toward the Cramer–Rao lower bound (CRLB – the smallest achievable mean squared error (MSE) with unbiased estimators) [71,72]. Surprisingly, roots of this refinement strategy could be found almost 30 years ago [73–76]. Main difference is in fact that the refinement strategy in the QML is applied to linear transform (STFT) based estimates instead of the nonlinear transforms results from the PD techniques. In this way, the refinement strategy is applied to the biased signal parameters estimates that are robust to noise influence. It means that the STFT related estimators have lower signal-to-noise ratio (SNR) threshold with respect to the PD ones and that the main goal of the refinement is to reduce the bias. However, removing bias and handling noise influence in the TF representations is tricky problem since for narrow windows the STFT (and other representations) has emphatic noise influence. For wider windows, the bias can be so large that the refinement cannot produce accurate results. Hence, multiple STFTs with different window lengths are calculated and sets of estimates are obtained for all of them. Final estimates are selected by maximization the ML criterion function. Combining robustness of the STFT to the noise influence, O'Shea refinement and ML criterion function, the QML has excellent performance surpassing all existing PD alternatives. Up to now, this technique has achieved several important extensions. It can be used for signals different from the PPS model, two-dimensional (2-D) signals [77], signals corrupted by impulsive noise [78], and mc PPSs [64]. It is extended to signals sampled bellow sampling rate [79] and it is combined with the random sample consensus (RANSAC) algorithm in order to further improve the SNR operational range [80].

This overview article of, in our opinion, important development in the field, is organized as follows. Signal model and related PD estimators are presented in Section 2. In Section 3, the QML steps are reviewed followed by the algorithm extensions. Numerical examples demonstrating advantages of the QML with respect to al-