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A family of gain-combined proportionate adaptive filtering algorithms for sparse system identification

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A B S T R A C T

The classical proportionate adaptive filtering (PAF) algorithms achieve a fast initial convergence for sparse impulse response. But the small coefficients receive very little gain so that the time needed to reach steady-state misalignment is increased. In addition, the PAF algorithms converge much slower than the original adaptive filtering (OAF) algorithms when the impulse response is dispersive. In order to address these problems, this paper proposes a family of gain-combined PAF (GC-PAF) algorithms. The gain-combined matrix of the proposed GC-PAF algorithms is implemented by using a sigmoidal activation function to adaptively combine the proportionate matrix and identity matrix, which can retain the advantages of both the PAF algorithms in the context of sparse impulse response and the OAF algorithms in the context of dispersive impulse response. Meanwhile, to be also applicable to the family of sign algorithms against impulsive noise, a general framework for the update of the sigmoidal activation function is obtained by using the gradient descent method to minimize the L_1-norm of the system output error. Simulations in the contexts of three different sparsity impulse responses have shown that the proposed GC-PAF algorithms perform much better than the OAF, PAF and improved PAF (IPAF) algorithms.

1. Introduction

The least-mean-square (LMS) and normalized LMS (NLMS) algorithms are the most popular adaptive filtering algorithms owing to their low computational complexity and ease of implementation [1]. However, the LMS- and NLMS-type algorithms often fail in terms of the convergence rate for the wide range of applications such as acoustic echo cancellation (AEC), network echo cancellation (NEC), room impulse response (RIR) identification, as well as relative transfer function (RTF) identification. The impulse responses of most of these applications are sparse in nature. Thus, sparsity property has long been investigated to improve the performance of the LMS and NLMS algorithms, such as the smoothed l_0-norm LMS (SL0-LMS) [2], non-uniform norm constraint LMS (NNCLMS) [3], and proportionate NLMS (PNLMS) [4] algorithms. Representatively, the PNLMS algorithm adjusts the adaptation gain in proportion to the estimated filter coefficient. But, it is difficult to *a priori* know the sparseness of the impulse response in real-world scenarios. Over the past nearly two decades, many variants of PNLMS algorithm have been proposed in the literatures [5–24]. A general framework for the proportionate-type NLMS algorithms is proposed in [22–24]. Some new proportionate NLMS algorithms can be derived following this framework.

The PNLMS algorithm was directly extended to the affine projection algorithm (APA) [25] and affine projection sign algorithm (APSA) [26] and thus yields the proportionate APA (PAPA) [14] and real-coefficient proportionate APSA (RP-APSA) [21], respectively. In [22] and [23], a general framework for the proportionate-type APA algorithms is derived. The proportionate adaptive filtering (PAF) algorithms including the PNLMS, adaptive segmented PNLMS (ASPNLMS) [22], PAPA and RP-APSA achieve a fast initial convergence rate in sparse impulse response. But the small coefficients receive very little gain so that the time needed to reach steady-state misalignment is increased. In addition, the PAF algorithms converge much slower than the original adaptive filtering (OAF) algorithms including the NLMS, APA and APSA when the impulse response is dispersive. Thus, the improved PAF (IPAF) algorithms including the improved PNLMS (IPNLMS) [6], variable parameter IPNLMS (VP-IPNLMS) [12], improved PAPA (IPAPA) [16] and real-coefficient improved proportionate APSA (RIP-APSA) [21] have been developed to address these problems. In addition, a family of block-sparse proportionate algorithms has also been proposed in [27–29] to address these problems above, which is obtained by optimizing a mixed $L_{2,1}$ norm of the weight coefficients of the adaptive filter. Today, it still is one of the most active topics in adaptive filtering to further investigate these issues. Inspired by

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[30], this paper proposes a family of gain-combined PAF (GC-PAF) algorithms, which refer to the GC-PNLMS algorithm, GC-PAPA and GC-PAPSA.

The gain-combined matrix of the proposed GC-PAF algorithms is implemented by using a sigmoidal activation function to adaptively combine the proportionate matrix and identity matrix, which can retain the advantages of both the PAF algorithms in the context of sparse impulse response and the OAF algorithms in the context of dispersive impulse response. Meanwhile, to obtain a general framework, the sigmoidal activation function is indirectly updated by minimizing the L_1-norm of the system output error, i.e., the algorithms should be robust against impulsive noise when this framework is applied to the family of sign algorithms. Benefiting from this combination approach, the small coefficients receive more gain and as a result the proposed algorithms to reach steady-state misalignment are faster. The performance of the proposed GC-PAF algorithms is tested in the context of three different sparsity impulse responses. Simulation results show that the proposed GC-PAF algorithms perform better than the OAF, PAF and IPAF algorithms.

The organization of this paper is as follows. The classical PNLMS algorithm is reviewed in Sec. 2. In Sec. 3, the framework of the GC-PAF algorithms is developed. In Sec. 4, the proposed framework is applied to the NLMS algorithm, APA and APSA, respectively. The simulation results and detail descriptions are presented in Sec. 5. In Sec. 6, the optimal value of the key parameter ρ_{gc} of the combined gain of the proposed algorithm is evaluated. Sec. 7 makes a conclusion.

2. Review of the PNLMS algorithm

We consider reference data $\{d(k)\}$ that arises from the linear model

$$d(k) = \mathbf{u}^T(k)\mathbf{w}_{opt} + \upsilon(k), \tag{1}$$

where k stands for the time index, superscript T denotes vector or matrix transpose operation, $\mathbf{u}(k) = [u(k), u(k-1), \ldots, u(k-L+1)]^T$ denotes an $L \times 1$ input vector with L being the filter length, $\mathbf{w}_{opt} = [\omega_0, \omega_1, \ldots, \omega_{L-1}]^T$ is an unknown weight vector that needs to be estimated, and $\upsilon(k)$ stands for the background noise.

The PNLMS algorithm is summarized by the following equations [5]:

$$e(k) = d(k) - \mathbf{u}^T(k)\mathbf{w}(k-1), \tag{2}$$

$$\mathbf{w}(k) = \mathbf{w}(k-1) + \mu \frac{\mathbf{G}(k)\mathbf{u}(k)e(k)}{\mathbf{u}^T(k)\mathbf{G}(k)\mathbf{u}(k) + \beta}, \tag{3}$$

$$\mathbf{G}(k) = \mathrm{diag}\{g_0(k), g_1(k), \ldots, g_{L-1}(k)\}, \tag{4}$$

where $e(k)$ is the system output error, $\mathbf{w}(k)$ is the estimate of $\mathbf{w}_{opt}$ at iteration k, μ is a global step size, β is the regularization factor, and $\mathbf{G}(k)$ is an $L \times L$ diagonal gain distribution matrix that determines the step sizes of the individual coefficients of the filter. For the PNLMS algorithm, the diagonal elements of $\mathbf{G}(k)$ are calculated as follows:

$$l_{max}(k-1) = \max\{|\omega_0(k-1)|, \ldots, |\omega_{L-1}(k-1)|\}, \tag{5}$$

$$\varphi_n(k) = \max\{\rho \max[\eta, l_{\max}(k-1)], |\omega_n(k-1)|\}, \tag{6}$$

$$g_n(k) = \frac{\varphi_n(k)}{\frac{1}{L}\sum_{i=0}^{L-1}\varphi_i(k)} \quad 0 \le n \le L-1 \tag{7}$$

where ρ and η are positive parameters with typical values $\rho = 5/L$ and $\eta = 0.01$, respectively. The parameter η regularizes the updating when all of the coefficients are zero at initialization, and the parameter ρ prevents the very small coefficients from stalling.

3. The framework of the proposed GC-PAF algorithms

The update equations of the weight vectors of the OAF algorithms (including the NLMS algorithm, APA and APSA) can be wrote as [1,22,25,26]:

$$\mathbf{w}(k) = \mathbf{w}(k-1) + \frac{1}{2}\mathbf{A}(k)\boldsymbol{\delta}(k), \tag{8}$$

where $\delta(k)$ is a Lagrange multiplier (vector), which can be derived by substituting (8) into the constraint condition of the corresponding algorithms, and $\mathbf{A}(k)$ is an input vector or matrix.

3.1. Algorithm design

In order to overcome the drawbacks of the PAF algorithms mentioned in Sec. 1, the weight vector $\mathbf{w}(k)$ of the adaptive filter is investigated by dividing it into two parts $\mathbf{w}_1(k)$ and $\mathbf{w}_2(k)$, one part $\mathbf{w}_1(k)$ retains the advantages of the PAF algorithms in the context of sparse impulse response, and the other part $\mathbf{w}_2(k)$ retains the advantages of the OAF algorithms in the context of dispersive impulse response. Then $\mathbf{w}_1(k)$ and $\mathbf{w}_2(k)$ are combined together by a variable mixing factor $\lambda(k)$ as

$$\mathbf{w}(k) = \lambda(k)\mathbf{w}_1(k) + (1-\lambda(k))\mathbf{w}_2(k), \tag{9}$$

where $0 \le \lambda(k) \le 1$. When $\lambda(k) = 1$, (9) reverts to the PAF algorithms; when $\lambda(k) = 0$, (9) becomes the OAF algorithms. Benefiting from this combination approach, the small coefficients receive more gain and as a result the proposed algorithms to reach steady-state misalignment are faster.

In order to deduce the update equation of the weight vector of the proposed algorithms, we assume that the weight vector $\mathbf{w}(k-1)$ is updated as $\mathbf{w}_1(k)$ by using the optimization criterion in [22], and is updated as $\mathbf{w}_2(k)$ by using (8), respectively, as

$$\mathbf{w}_1(k) = \mathbf{w}(k-1) + \frac{1}{2}\mathbf{G}(k)\mathbf{A}(k)\boldsymbol{\delta}(k), \tag{10}$$

$$\mathbf{w}_2(k) = \mathbf{w}(k-1) + \frac{1}{2}\mathbf{A}(k)\boldsymbol{\delta}(k). \tag{11}$$

Substituting (10) and (11) into (9), we can obtain the update equation of the weight vector as

$$\begin{aligned}
\mathbf{w}(k) &= \lambda(k)\mathbf{w}_1(k) + (1-\lambda(k))\mathbf{w}_2(k) \\
&= \lambda(k)\left\{\mathbf{w}(k-1) + \frac{1}{2}\mathbf{G}(k)\mathbf{A}(k)\boldsymbol{\delta}(k)\right\} \\
&\quad + (1-\lambda(k))\left\{\mathbf{w}(k-1) + \frac{1}{2}\mathbf{A}(k)\boldsymbol{\delta}(k)\right\} \\
&= \mathbf{w}(k-1) + \frac{1}{2}\lambda(k)\mathbf{G}(k)\mathbf{A}(k)\boldsymbol{\delta}(k) + \frac{1}{2}(1-\lambda(k))\mathbf{A}(k)\boldsymbol{\delta}(k) \\
&= \mathbf{w}(k-1) + \frac{1}{2}\{\lambda(k)\mathbf{G}(k) + (1-\lambda(k))\mathbf{I}\}\mathbf{A}(k)\boldsymbol{\delta}(k) \\
&= \mathbf{w}(k-1) + \frac{1}{2}\mathbf{G}_{\mathbf{I}}(k)\mathbf{A}(k)\boldsymbol{\delta}(k),
\end{aligned} \tag{12}$$

where $\mathbf{I}$ is an $L \times L$ identity matrix, and

$$\mathbf{G}_{\mathbf{I}}(k) = \lambda(k)\mathbf{G}(k) + (1-\lambda(k))\mathbf{I}. \tag{13}$$

3.2. Variable mixing factor design

Inspired by the convex combination method [31], the variable mixing factor $\lambda(k)$ is defined as the output of a sigmoidal activation function [32,33] with an intermediate variable $\alpha(k)$ as

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