Digital Signal Processing ••• (••••) •••–•••

Contents lists available at ScienceDirect

Digital Signal Processing

www.elsevier.com/locate/dsp

A robust high-resolution time–frequency representation based on the local optimization of the short-time fractional Fourier transform

Md. Abdul Awal [a,b,*], Samir Ouelha [c], Shiying Dong [a,b], Boualem Boashash [a,b,c]

[a] The University of Queensland, UQ Centre for Clinical Research, Brisbane, Australia
[b] The University of Queensland, Perinatal Research Centre, School of Medicine, Brisbane, Australia
[c] Department of Electrical Engineering, Qatar University, Doha, Qatar

ARTICLE INFO

Article history:
Available online xxxx

Keywords:
Energy concentration
Fractional S-method (FrSM)
Instantaneous frequency (IF) estimation
Local optimization
Short-time fractional Fourier transform (STFrFT)
Quadratic time–frequency distribution (QTFD)

ABSTRACT

The Locally Optimized Spectrogram (LOS) defines a novel method for obtaining a high-resolution time–frequency (t, f) representation based on the short-time fractional Fourier transform (STFrFT). The key novelty of the LOS is that it automatically determines the locally optimal window parameters and fractional order (angle) for all signal components, leading to a high-resolution and cross-terms free time–frequency representation. This method is suitable for multicomponent and non-stationary signals without *a priori* signal information. Simulated signals, real biomedical applications, and various measures are used to validate the improved performance of the LOS and compare it with other state-of-the-art methods. The robustness of the LOS is also demonstrated under different signal-to-noise ratio (SNR) conditions. Finally, the relationship between the LOS and other time–frequency distributions (TFDs) is depicted and a recursive formulation is presented and shows the trade-off between the cross-terms suppression and auto-terms resolution.

1. Introduction

As most signals observed in nature are non-stationary, it is useful to represent them in a time–frequency (t, f) domain. One of the simplest ways is to use the spectrogram, as it is robust, computationally efficient and easy to interpret because it is cross-terms free when components are not too close (see section 4.2 in [4]). It is still one of the prominent methods for (t, f) analysis in many applications such as speech processing [1], signal denoising [2] and instantaneous frequency (IF) estimation [3]. However, the performance of the spectrogram is heavily dependent on the analysis window which is set heuristically [4, pp. 78–79] [5]. This fixed window approach is not suitable for multicomponent signals composed of both short and long duration overlapping components. This is a major limitation of the use of the spectrogram. A solution is to optimize the analysis window locally to produce an improved (t, f) representation [5].

A variant of the spectrogram is the S-transform whose window (Gaussian) width is inversely proportional to the frequency. It improves the time resolution at higher frequencies and the frequency resolution at lower frequencies [4, pp. 310–311]. There is no parameter that is used to adjust the analysis window in the standard S-transform. A modified S-transform (MST) [6] can be defined in a number of ways e.g. by optimizing the window length using energy concentration [7]. A frequency based window-width optimization has also been proposed to improve the resolution of the S-transform [8]. Another variant is the S-method (SM) [9, pp. 256–261] and the performance of this method also depends on the choice of analysis window, window length and correction terms which are needed to be set and optimized. In addition, signals composed of closely spaced chirp signals, or a mixture of short and long duration components which overlap within the (t, f) domain pose significant problems for these modified methods because the representation of the signal components is inevitably compromised by the lack of local optimization. To overcome these issues, this study proposes an improved (t, f) representation, named Locally Optimal Spectrogram (LOS) based on the fractional Fourier transform (FrFT) to locally enhance the resolution. This method is well-suited for the analysis of multicomponent, non-stationary signals lacking *a priori* information, a common situation when one deals with real-life signals such as electroencephalograph (EEG) signals.

This paper presents a methodology to get the new LOS from the short-time fractional Fourier transform (*STFrFT*) by locally optimizing both window length and chirp rate. The results demonstrate the effectiveness of the LOS using different simulated signals and

* Corresponding author at: SPRC Group, Level 4, UQCCR, Brisbane, QLD-4029, Australia.
E-mail addresses: m.awal@uq.edu.au, awalece04@yahoo.com (Md.A. Awal).

http://dx.doi.org/10.1016/j.dsp.2017.07.022
1051-2004/

a real-life application which uses clinical EEG signals. The contributions and key topics covered by this paper are as follow:

- A simple and efficient optimization procedure is proposed to enhance the resolution of the spectrogram for non-stationary and multicomponent signals by taking the window length and chirp rate into account. This process ensures the compact representation of the local signal behavior both in time and frequency (Section 2).
- Different simulated multicomponent signals of varied amplitude and wide-ranging (t, f) characteristics are used to demonstrate the efficiency of the proposed method. The LOS is compared with other state-of-the-art fixed and adaptive time–frequency (t, f) methods in Section 4.
- Various (t, f) measure indexes are used to ensure: (1) a thorough and rigorous quantitative evaluation and (2) a fair comparison of the LOS with other state-of-the-art methods in Section 5.
- The robustness of the LOS is evaluated for instantaneous frequency (IF) estimation purpose. In Section 6, a simulation is run 100 times under different SNR conditions (Monte-Carlo approach) comparing the effectiveness and robustness of the LOS with other well-known TFDs.
- Section 7 demonstrates the superior performance of the LOS method on a real application that is to detect clinically abnormal burst-suppression patterns in neonatal EEG signals.
- Section 8 discusses a relationship between LOS and other TFDs; this relationship generalizes the proposed approach.

Note that, this paper focuses only on the analysis of QTFDs due to their simple interpretation, high resolution and widespread use. Appendix A defines state-of-the-art QTFDs. Appendix B provides a pseudocode for generating the fractional S-Method. In addition, computer programs used in this study are described in Appendix C.

2. Background literatures and existing works

The proposed LOS is derived from FrFT and therefore, it is important to briefly describe the FrFT before the presentation of the proposed method.

2.1. Fractional Fourier transform (FrFT)

The FrFT is the generalization of the classical Fourier transform (FT). It can be regarded as a rotation by an arbitrary angle α in the (t, f) plane [10,11]. The FT corresponds to a rotation over an angle $\alpha = \pi/2$ in the (t, f) plane. The FrFT is defined as [12]:

$$X_\alpha(u) = \int_{-\infty}^{\infty} x(t) K_\alpha(u,t) dt \tag{1}$$

where $\alpha = p(\pi/2)$; $p \in \mathbb{R}$ and the kernel K_α is defined by

$$K_\alpha(u,t) = \begin{cases} \sqrt{\frac{1-j\cot\alpha}{2\pi}} e^{j\frac{(u^2+t^2)\cot\alpha}{2} - \frac{jut}{\sin\alpha}}, & \alpha \neq q\pi \\ \delta(u-t), & \alpha = 2q\pi \\ \delta(u+t), & \alpha = (2q+1)\pi \end{cases}, \quad q \in \mathbb{N} \tag{2}$$

where δ denotes the Dirac function.

Special cases of the FrFT are: $X_0(u) = x(t)$, $X_\pi(u) = x(-t)$ and $X_{\pi/2}(u)$ corresponding to the classical FT. The orthogonal pair (u, v) characterizes a new physical quantity in the fractional Fourier domain and related to (t, f) as [13]:

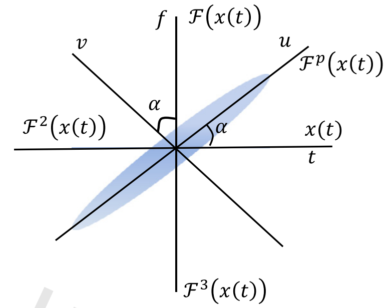

Fig. 1. Illustration of classical FT and Fractional FT and the representation of a signal whose principal axis corresponds to fractional time–frequency axis. α is the angle $(\alpha = p(\pi/2))$ and p is the transform order ranging from 0 to 4.

$$\begin{pmatrix} t \\ f \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} \tag{3}$$

where u, v represents the rotation of time and frequency respectively, with an angle α, in the fractional domain. Therefore, the (u, v) plane is only the rotation of the (t, f) plane in the fractional domain by an angle α (see Fig. 1).

2.2. Rationale for using the FrFT

Fig. 1 interprets the concept of FrFT in the (t, f) plane. By applying the classical FT denoted by, $\mathcal{F}(x(t))$, a time domain signal $x(t)$ is changed to its frequency domain counterpart, $X(f)$, which rotates the signal over an angle $\pi/2$ counter-clockwise. By again applying FT i.e. $\mathcal{F}^2(x(t)) = x(-t)$ rotates $2\pi/2$ angle and similarly $\mathcal{F}^3(x(t)) = X(-f)$ rotates $3\pi/2$ and $\mathcal{F}^4(x(t)) = x(t)$ rotates $4\pi/2$ [12].

The FrFT $\mathcal{F}^p(x(t))$ provides a generalization of the classical FT and offers improved flexibility when designing high resolution (t, f) signatures as the signal chirp rate can be adapted by this approach. In the case of a signal whose principal axis does not correspond to the time or the frequency plane, as in Fig. 1, the FrFT applies an affine transformation in the phase plane leading to an optimum signal representation [13]. This can be done by properly adjusting the FrFT transform order (angle). This justifies the analysis of a signal in the fractional Fourier domain.

2.3. Short-time fractional Fourier transform (STFrFT)

By generalizing the short-time Fourier transform (STFT) in the same manner as the FT, the short-time fractional Fourier transform (STFrFT) can be defined as [9, pp. 135–136]:

$$\mathrm{STFrFT}_\alpha(u,v) = \int_{-\infty}^{\infty} X_\alpha(u+\tau) w^*(\tau) e^{-j2\pi v\tau} d\tau \tag{4}$$

$$\mathrm{STFrFT}_\alpha(u,v) = e^{j\pi(uv-tf)} \int_{-\infty}^{\infty} x(t+\tau) \mathcal{F}^{-\alpha}\big(w(\tau)\big) e^{-2j\pi\tau f} d\tau \tag{5}$$

where $w(\tau)$ is a window function. These formulations indicate that the lag truncation can be applied prior to or after signal rotation with the same results [9, pp. 135–136]. The fractional spectrogram (FrSpec) is calculated by squaring the magnitude of STFrFT i.e.

$$\mathrm{FrSpec}(u,v) = \left|\mathrm{STFrFT}_\alpha(u,v)\right|^2 \tag{6}$$

Eqns. (4)–(6) indicate that the performance of the STFrFT and FrSpec is determined by the rotation angle α, the shape and the length of the analysis window. The transform order determines

Please cite this article in press as: Md.A. Awal et al., A robust high-resolution time–frequency representation based on the local optimization of the short-time fractional Fourier transform, Digit. Signal Process. (2017), http://dx.doi.org/10.1016/j.dsp.2017.07.022